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Reevaluation of Regulatory Guidance on Modal Response Combination Methods for Seismic Response Spectrum Analysis

Brookhaven National Laboratory

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ABSTRACT

Regulatory Guide 1.92, "Combining Modal Responses and Spatial Components in Seismic Response Analysis," was last revised in 1976. The objectives of this project were to re-evaluate the current regulatory guidance for combining modal responses in response spectrum analysis; evaluate recent technical developments; and recommend revisions to the regulatory guidance. In addition, Standard Review Plan Section 3.7.2, "Seismic System Analysis," was reviewed to identify related sections which may need to be revised. The objectives were addressed through a literature review of past studies, supplemented by analysis of a piping system model previously utilized in NUREG/CR-5627, "Alternate Modal Combination Methods in Response Spectrum Analysis."

This project evaluated (1) methods for separation of the in-phase and out-of-phase modal response components; (2) methods for combination of the out-of-phase modal response components; (3) the contribution of "missing mass"; and (4) the combination of the three elements of response to produce the total response. Numerical results from response spectrum analyses were compared to corresponding time history analysis results to assess the accuracy of the various combination methods tested.

During the course of the project, several insights relating to potential improvements in the methodology for seismic analysis were identified and documented. These include (1) improvements in correlation between mode superposition time history and direct integration time history; (2) use of response spectrum generation single degree of freedom oscillator responses to define the frequency above which modal responses are in-phase with the input time history; and (3) evaluation of the effects of potential differences in mass distribution used in static and dynamic analyses of a piping system.

The conclusions of the project are (1) several of the modal response combination methods evaluated should be included as acceptable methods in the next revision to Regulatory Guide 1.92; (2) deletion of several of the modal response combination methods currently in the Regulatory Guide should be considered; (3) the Regulatory Guide should be revised to specifically define acceptable procedures for constructing the total response spectrum analysis solution; and (4) several variations of modal response combination methods, which were examined to try to improve accuracy, showed promising results and warrant further study.

This project did not specifically analyze the modal response combination methods applied to highly flexible, low frequency systems. A separate evaluation for these systems should be considered.

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EXECUTIVE SUMMARY

Regulatory Guide 1.92 "Combining Modal Responses and Spatial Components in Seismic Response Analysis" was last revised in 1976. The objective of this project were to re-evaluate the current regulatory guidance for combining modal responses in response spectrum analysis; evaluate recent technical developments; and recommend revisions to the regulatory guidance. In addition, Standard Review Plan (SRP) Section 3.7.2, "Seismic System Analysis," was reviewed to identify related sections which may need to be revised. The objectives were addressed through a literature review of past studies, supplemented by analysis of a piping system model previously utilized in NUREG/CR-5627, "Alternate Modal Combination Methods in Response Spectrum Analysis".

The project evaluated (1) methods for separation of the in-phase and out-of-phase modal response components; (2) methods for combination of the out-of-phase modal response components; (3) the contribution of "missing mass"; and (4) the combination of the three elements of response to produce the total response. Numerical results from response spectrum analyses were compared to corresponding time history analysis results to assess the accuracy of the various combination methods tested.

The methods selected for evaluation were those which have been subjected to the greatest level of prior review and assessment. For separation of the in-phase and the out-of-phase modal response components, the methods proposed by Lindley-Yow, Hadjian and Gupta were evaluated. For combination of the out-of-phase modal response components, the Square Root of the Sum of the Squares (SRSS), NRC Grouping, NRC Ten Percent, Rosenblueth's Double Sum Combination (DSC), the NRC DSC, and Der Kiureghian's Complete Quadratic Combination (CQC) methods were evaluated. For treatment of the "missing mass" contribution, the method of Kennedy was evaluated. Response spectrum analyses were conducted by combining elements of the above methods to construct complete response spectrum analysis solutions.

Baseline time history solutions, utilizing both mode superposition and direct integration methods, were also generated. Excellent correlation between the two time history methods was obtained by adding the missing mass contribution to the mode superposition solution. Missing mass proved to be significant even though the mode superposition solution included 31 modes, up to 70 Hz.

Based on the literature review and the numerical results generated in this project, it is concluded that Rosenblueth's DSC and Der Kiureghian's CQC methods for combining out-of-phase modal response components, coupled with the Lindley-Yow or Gupta method for separation of the in-phase and the out-of-phase modal response components, and inclusion of Kennedy's missing mass contribution to account for modes with frequencies above that corresponding to the Zero Period Acceleration (ZPA) of the response spectrum, constitute the best methodologies currently available for response spectrum analysis.

EXECUTIVE SUMMARY

The NRC Grouping, NRC Ten Percent and NRC-DSC methods for combining out-of-phase modal response components produced more conservative but less accurate results. Removal of these methods from the Regulatory Guide should be considered, because absolute summation of closely spaced modal responses has no documented technical basis. SRSS remains applicable in the absence of closely spaced modes.

Separation of modal responses into in-phase and out-of-phase components for modes with frequencies below the ZPA frequency of the response spectrum produced more accurate results than commonly applied past methods, in which all modes below the ZPA frequency were considered to be out-of-phase. It is important to note that at low frequency (<frequency of the peak spectral acceleration) modal responses should be treated as out-of-phase. A limitation of the Lindley-Yow formulation is that low frequency modal responses are separated into out-of-phase and in-phase components; consequently, when significant low frequency modal responses exist, the Lindley-Yow formulation must be appropriately modified. The Gupta formulation correctly assumes low frequency modes are out-of-phase. This project did not specifically analyze modal response combination methods applied to systems with significant low frequency response.

At the request of the Project Peer Review Panel, an extensive investigation of individual modal response contributions to both the response spectrum and time history solutions was conducted for selected support reactions, in order to explain significant differences in prediction. Once the phenomenon was understood, several variations of the initial modal response combination methods were defined and additional response spectrum analyses were conducted. Improvement in accuracy was achieved. As part of this additional analytical effort, a methodology was defined to utilize the SDOF oscillator responses from the response spectrum generation analysis to establish the frequency above which modal responses are in-phase with the input time history. This technique should be applicable to all response spectra.

During the course of the project, static analysis results for the mass times the ZPA were generated utilizing system mass distributions developed by both the dynamic analysis option and the static analysis option of the same computer code (based on SAP V). Significant differences in solution were noted, which are attributed to differences in the treatment of mass. In the dynamic analysis model, mass is apportioned to node points; in the static analysis model, mass is treated as distributed along the element length. While this situation may not exist in all piping analysis codes, it is important to verify that consistent static analysis results are obtained for both dynamic and static mass distributions. Lack of consistency usually indicates that the element breakup in the piping model is not sufficiently refined to accurately predict support reactions and internal bending moments.

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ABBREVIATIONS

ASCE	American Society of Civil Engineers
BNL	Brookhaven National Laboratory
CQC	Complete Quadratic Combination
DSC	Double Sum Combination
MODF	Multiple Degree of Freedom
NRC	Nuclear Regulatory Commission
RB-DSC	Rosenblueth's Double Sum Combination
RG	Regulatory Guide
RSA	Response Spectrum Analysis
SDOF	Single Degree of Freedom
SRP	Standard Review Plan
SRSS	Square Root of the Sum of the Squares
ZPA	Zero Period Acceleration

1 INTRODUCTION

General Design Criterion 2, "Design Basis for Protection Against Natural Phenomena" of Appendix A, "General Criteria for Nuclear Power Plants" to 10CFR Part 50, "Licensing of Production and Utilization Facilities" requires that nuclear power plant structures, systems, and components which are important to safety be designed to withstand the effects of earthquakes without loss of capability to perform their safety functions. In addition, Paragraph (a)(1) of Section VI of Appendix A, "Seismic and Geologic Siting Criteria for Nuclear Power Plants" to 10CFR Part 100, "Reactor Site Criteria" identifies the use of a suitable dynamic analysis as one method of ensuring that structures, systems, and components can withstand seismic loads. Related requirements for new license applicants (after January 10, 1997) are specified in Paragraph (a) (1) of Section IV of Appendix S, "Earthquake Engineering Criteria for Nuclear Power Plants" to 10 CFR Part 50.

The United States Nuclear Regulatory Commission (NRC) issues Regulatory Guides (RG) which describe methods acceptable to the NRC staff for satisfying regulations. One such guide is RG 1.92, "Combining Modal Responses and Spatial Components in Seismic Response Analysis," (Reference 1). This guide was last revised in 1976, prior to a number of significant technical developments for combining modal responses.

The 1989 revision to Standard Review Plan (SRP) Section 3.7.2, "Seismic System Analysis," (Reference 2) recognized a number of recent technical developments by reference, and stated that their application to nuclear power plant seismic analysis is subject to review on a case-by-case basis. Also incorporated into SRP Section 3.7.2 as Appendix A was a procedure developed by Kennedy (Reference 3) to address high frequency mode effects.

The objective of this project was to evaluate these recent developments for modal response combination, through a literature review and analytical effort, and to provide recommendations for revision of RG 1.92 and SRP Section 3.7.2 which reflect the current state of technology for combining modal responses in seismic response spectrum analysis..

The design of structures, systems, and components for seismic loads is complicated by the uncertainty about the future seismic event. For seismic analysis where an accurate record of the seismic input exists, time history analysis is the best approach to mathematical prediction of seismic structural response. In the seismic design process, two primary approaches are available to account for the uncertainty in the seismic ground motion: perform a number of time history analyses utilizing an appropriate set of acceleration records or perform response spectrum analysis utilizing a bounding peak acceleration vs. frequency spectrum. Either approach, properly implemented, should provide a conservative basis for seismic design.

A significant feature of response spectrum analysis is that only the maximum dynamic response is predicted. In time history analysis, the response vs. time is predicted. The mathematical simplicity of response spectrum analysis is achieved by calculating the independent peak modal responses and then applying a rule for combining these, in order to predict the peak dynamic response. The input response spectrum defines the acceleration to be applied to each natural mode of vibration of the structure, depending on its modal frequency. However, the spectrum provides no information about the time phasing of excitation of these modes.

It is evident that the modal response combination method used to predict the peak dynamic response is a critical element of the response spectrum analysis method. Many researchers have studied this specifically for seismic analysis, and a number of increasingly sophisticated methods have been developed. In 1984, Kennedy (Reference 3) reviewed alternate methods for modal response combination for the NRC and provided recommendations for revision to regulatory guidance. Gupta (Reference 4, Chapter 3) provides an excellent review of these developments up to the late 1980's, including his own modal response combination method. In 1994, Mertens (Reference 5) presented a refinement of previously developed methods for combining amplified modal responses.

This present investigation does not address the issue of seismic design methodology - time history analysis vs. response spectrum analysis. It is focused on comparison and evaluation of different modal response combination methods for use in response spectrum analysis. Because the objective of response spectrum analysis is to predict, with reasonable accuracy, the peak dynamic response to a time varying acceleration input, comparison to time history solutions is the primary method employed to evaluate the applicability and limitations of the various modal response combination methods.

The initial phase of this program focused on review of the technical literature and selection of candidate modal response combination methods for more detailed evaluation. Acceptable methods in RG 1.92 were also included to provide a comparison to more recent technical developments. References 3 and 4 provided an excellent starting point. In addition, prior numerical studies conducted by Brookhaven National Laboratory (References 6, 7, 8) on response spectrum analysis of piping systems provided a quantitative database for alternate modal response combination methods. Industry standards such as ASCE Standard 4 (References 9, 10) were also reviewed. The work of Maison et al (Reference 11) provided a numerical study of different modal response combination methods applied to a building structure. A significant body of additional reference material was included in the literature review.

The methods selected for evaluation were those which have been subjected to the greatest level of prior review and assessment. The evaluation addressed (1) methods for combination of the out-of-phase modal response components; (2) methods for separation of the in-phase and out-of-phase modal response components; (3) the contribution of high frequency modal responses; and (4) combination of the three elements of response to produce the total response. The term "in-phase" denotes the response component that is in-phase with the time varying input acceleration; the term "out-of-phase" denotes the response component that is not in-phase with the time varying input acceleration. Gupta (Reference 4) refers to these as the "rigid" or "pseudo-static" response and the "damped periodic" response, respectively.

It is important to note that individual modal responses which are each "out-of-phase" with respect to the time varying input acceleration may be nearly in-phase with each other. This is commonly referred to as the "closely spaced modes" issue, because modes close in frequency are likely to respond nearly in-phase with each other when excited by the same time varying input acceleration. This is addressed by methods for combination of the "out-of-phase" modal responses.

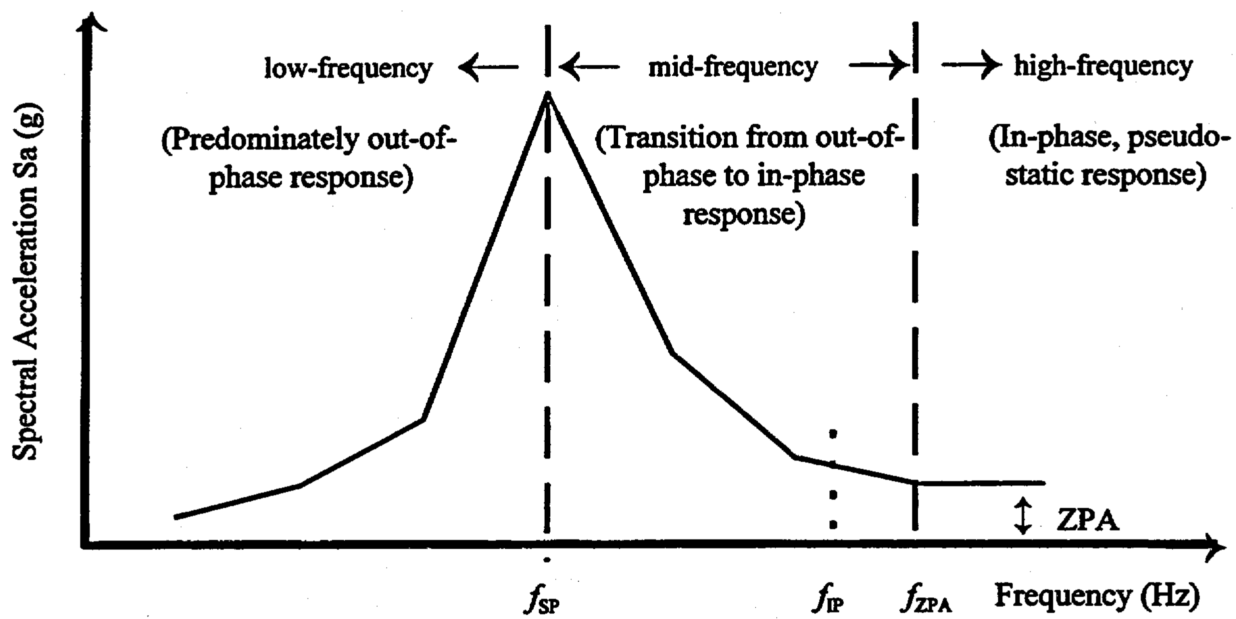
Modes with frequencies higher than the frequency (f_{ZPA}) at which the spectral acceleration returns to the Zero Period Acceleration (ZPA) respond pseudo-statically to the time varying input acceleration and, therefore, are in-phase with each other. The contribution of these modes to the total response is most accurately and efficiently treated by static analysis of the "missing mass" (i.e., system mass not participating in the modes with frequencies below f_{ZPA}) multiplied by the ZPA. Modes with frequencies in the amplified region of the response spectrum ($f < f_{ZPA}$) generally have both an in-phase component and an out-of-phase component. The in-phase modal response components and the missing mass contribution are combined algebraically, to produce the total in-phase response component.

For combination of the out-of-phase modal response components, Square Root of the Sum of the Squares (SRSS), NRC Grouping, NRC Ten Percent, Rosenblueth's DSC (Reference 12), NRC-DSC, and Der Kiureghian's CQC (Reference 13) methods were evaluated. For separation of the in-phase and the out-of-phase modal response components, the methods proposed by Lindley and Yow (Reference 14), Hadjian (Reference 15), and Gupta (Reference 4) were evaluated. For treatment of the missing mass contribution, the method of Kennedy (Reference 3) was evaluated.

2 DESCRIPTION OF MODAL RESPONSE COMBINATION METHODS

To lay the groundwork for the ensuing discussions of modal response combination methods, it is necessary to define the different regions of a typical seismic response spectrum and key frequencies which divide these regions. The major application of seismic response spectrum analysis in the nuclear industry is for systems and components attached to building structures.

A building-filtered in-structure response spectrum depicting spectral acceleration vs. frequency is the typical form of seismic input for such analyses. This type of spectrum usually exhibits a pronounced peak at the fundamental frequency of the building/soil dynamic system. An idealized in-structure response spectrum is shown below; the spectral regions and key frequencies are indicated.



f_{SP} = frequency at which the peak spectral acceleration is reached; typically the fundamental frequency of the building/soil system

f_{ZPA} = frequency at which the spectral acceleration returns to the zero period acceleration (ZPA)

f_{IP} = frequency above which the SDOF modal responses are considered to be in-phase with the time varying input acceleration used to generate the response spectrum

The high frequency region of the spectrum ($> f_{ZPA}$) is characterized by no amplification of the peak acceleration of the input time history. A SDOF oscillator having a frequency $> f_{ZPA}$ is accelerated in-phase and with the same acceleration magnitude as the applied acceleration, at each instant in time. A system or component with fundamental frequency $> f_{ZPA}$ is correctly analyzed as a static problem subject to a loading equal to its mass times the ZPA. The system or component is said to respond "pseudo-statically." This concept can be extended to the high frequency ($> f_{ZPA}$) modal responses of multi-modal systems or components. The mass not participating in the amplified modal responses (i.e., "missing mass") multiplied by the ZPA is applied in a static analysis, to obtain the response contribution from all modes with frequencies $> f_{ZPA}$.

In the low-frequency region of the spectrum ($< f_{SP}$), the modal responses of SDOF oscillators are not in-phase with the applied acceleration time history, and generally are not in-phase with each other. These are designated "out-of-phase" modal responses. Since a response spectrum provides only peak acceleration vs. frequency, with no phasing information, the out-of-phase peak modal responses for a multi-modal structural system requires a rule or methodology for combination. Based on the assumption that the peak modal responses are randomly phased, the square root of the sum of the squares (SRSS) method was developed and adopted. Modifications to the SRSS method were subsequently developed, in order to account for potential phase correlation when modal frequencies are numerically close (i.e., "closely spaced modes").

In the mid-frequency region (f_{SP} to f_{ZPA}), it has been postulated that the peak SDOF oscillator modal responses consist of two distinct and separable elements. The first element is the out-of-phase response component and the second element is the in-phase response component. It is further postulated that there is a continuous transition from out-of-phase response to in-phase response. If $f_{IP} < f_{ZPA}$ can be defined, then the mid-frequency region can be further divided into two sub-regions: $f_{SP} < f < f_{IP}$ and $f_{IP} \leq f \leq f_{ZPA}$.

It is noted that past practice in the nuclear power industry has been to assume that individual modal responses in the mid-frequency region ($f_{SP} < f < f_{ZPA}$) are out-of-phase, and that the combination methods applicable to the low-frequency region are also applicable to the mid-frequency region.

Three elements are needed to define a suitable methodology for the mid-frequency region:

1) A definition for f_{IP}.
2) A method for separating the in-phase and out-of-phase components of individual peak modal responses.
3) A phase relationship for combining the total out-of-phase response component with the total in-phase response component.

Methods for modal response combination are described in the following sections, where the following notation is used:

Sa_i = Spectral Acceleration for mode i
R_i = Response of mode i
α_i = In-phase response ratio for mode i
Rr_i = In-phase response component for mode i
Rp_i = Out-of-phase response component for mode i
Rr = Total in-phase response component from all modes
Rp = Total out-of-phase response component from all modes
Rt = Total combined response from all modes
C_{jk} = Modal response correlation coefficient between modes j and k.

2.1 Combination of Out-of-Phase Modal Response Components

In generalized form, all of the out-of-phase modal response combination methods can be represented by a single equation:

$$Rp = \left[\sum_{j=1}^{n} \sum_{k=1}^{n} C_{jk} \, Rp_j \, Rp_k \right]^{1/2} \qquad \text{(Eqn. 2 - 1)}$$

The mode correlation coefficients C_{jk} are uniquely defined for each method.

2.1.1 Square Root of the Sum of the Squares (SRSS) Method

At the foundation of all methods for combining uncorrelated modal responses is the SRSS method. All of the methods for combination of the out-of-phase response components are equivalent to SRSS if there are no “closely spaced” modes.

In this case,

$$\begin{aligned} C_{jk} &= 1.0 \text{ for } j = k \\ C_{jk} &= 0.0 \text{ for } j \neq k \end{aligned} \qquad \text{(Eqn. 2 - 2)}$$

and Equation 2-1 reduces to:

$$Rp = \left[\sum_{i=1}^{n} Rp_i^2 \right]^{1/2} \qquad \text{(Eqn. 2 - 3)}$$

2.1.2 NRC Grouping Method

The NRC Grouping Method (Reference 1) is the most commonly applied method of accounting for closely spaced modes in the nuclear power industry. The system modal responses are grouped and summed absolutely before performing SRSS combination of the groups. To illustrate the method, consider a system with 10 modes, with responses Rp_1 through Rp_{10}, and associated frequencies f_1 through f_{10} having the following distribution:

$$f_2 > 1.1 f_1$$

$$f_3, f_4 < 1.1 f_2$$

$$f_5 > 1.1 f_2$$

$$f_6 < 1.1 f_5$$

$$f_7 > 1.1 f_5$$

$$f_8 > 1.1 f_7$$

$$f_9, f_{10} < 1.1 f_8$$

The modal responses are grouped such that the lowest and highest frequency modes in a group are within 10% and no mode is in more than one group. Using the distribution above, the following groups are created:

$$GR_1 = Rp_1$$

$$GR_2 = |Rp_2| + |Rp_3| + |Rp_4|$$

$$GR_3 = |Rp_5| + |Rp_6|$$

$$GR_4 = Rp_7$$

$$GR_5 = |Rp_8| + |Rp_9| + |Rp_{10}|$$

$$Rp = \left[\sum_{i=1}^{n} GR_i^2 \right]^{1/2} \qquad \text{(Eqn. 2 - 4)}$$

In this illustration, n = 5.

The major criticism of the NRC Grouping Method is the use of absolute summation within each group. If modal responses are assumed to be correlated because they have closely spaced frequencies, then summation should be algebraic within each group. The bias toward conservatism in the NRC Grouping Method is somewhat contradictory to the basic premise for grouping.

When expressed in terms of the mode correlation coefficients C_{jk}, the NRC Grouping Method may be defined as follows:

$$\begin{aligned} C_{jk} &= 1.0 && \text{for } j = k \\ C_{jk} &= 0.0 && \text{for } j \neq k\text{, not in same group} \\ C_{jk} &= 1.0 && \text{for } j \neq k\text{, in the same group, } Rp_j \text{ and } Rp_k \text{ have same sign} \\ C_{jk} &= -1.0 && \text{for } j \neq k\text{, in the same group, } Rp_j \text{ and } Rp_k \text{ have opposite signs} \end{aligned} \qquad \text{(Eqn. 2 - 5)}$$

In implementing the NRC Grouping Method, the approach presented in the illustration is more straightforward.

2.1.3 NRC Ten Percent Method

The NRC Ten Percent Method (Reference 1) is a generally more conservative variation of the NRC Grouping Method. Closely spaced modes are defined as modes with frequencies within 10% of each other and absolute summation of the closely spaced modal responses is specified. The difference is that modal responses are not grouped.

In terms of the mode correlation coefficients, C_{jk}, the NRC Ten Percent Method is defined as follows:

$$
\begin{aligned}
C_{jk} &= 1.0 && \text{for } j = k \\
C_{jk} &= 0.0 && \text{for } j \neq k\text{, and } f_j \text{ and } f_k \text{ separated by} > 10\% \text{ of the lower frequency} \\
C_{jk} &= 1.0 && \text{for } j \neq k\text{, and } f_j \text{ and } f_k \text{ separated by} \leq 10\% \text{ of the lower frequency; } Rp_j \text{ and } Rp_k \text{ same sign} \\
C_{jk} &= -1.0 && \text{for } j \neq k\text{, } f_j \text{ and } f_k \text{ separated by} \leq 10\% \text{ of the lower frequency; } Rp_j \text{ and } Rp_k \text{ opposite signs}
\end{aligned}
\qquad \text{(Eqn. 2 - 6)}
$$

The definition of C_{jk} is analogous to that for the NRC Grouping Method, except that grouping is not performed.

As an illustration of the difference between these two methods, assume three modal responses Rp_1, Rp_2, Rp_3 with frequencies f_1, f_2, f_3 and a frequency distribution defined as follows:

$$f_2 \leq 1.1 f_1$$

$$1.1 f_1 < f_3 \leq 1.1 f_2$$

By the NRC Grouping Method,

$$Rp = \left[\sum_{i=1}^{2} GR_i^2 \right]^{1/2}$$

where

$$GR_1 = |Rp_1| + |Rp_2|$$

$$GR_2 = Rp_3$$

or

$$Rp = \left[\sum_{i=1}^{3} Rp_i^2 + 2\,|Rp_1 * Rp_2| \right]^{1/2}$$

By the NRC Ten Percent Method,

$$Rp = \left[\sum_{i=1}^{3} Rp_i^2 + 2\,|Rp_1 * Rp_2| + 2|Rp_2 * Rp_3| \right]^{1/2}$$

The NRC Ten Percent Method has an additional contribution to Rp because $f_3 \leq 1.1\ f_2$. The NRC Ten percent Method will always produce results ≥ the NRC Grouping Method.

2.1.4 Rosenblueth's Double Sum Combination (DSC)

Rosenblueth (Reference 12) provided the first significant mathematical approach to evaluation of modal correlation for seismic response spectrum analysis. It is based on the application of random vibration theory, utilizing a finite duration of white noise to represent seismic loading. A formula for calculation of the coefficients C_{jk} as a function of the modal circular frequencies (ω_j, ω_k), modal damping ratios (β_j, β_k), and the time duration of strong earthquake motion (t_D) was derived. Using the form of the equation from Reference 1,

$$C_{jk} = \frac{1}{1 + \left\{ \dfrac{\omega_j' - \omega_k'}{\beta_j' \omega_j + \beta_k' \omega_k} \right\}^2} \qquad \text{(Eqn. 2 - 7)}$$

$$\text{where} \quad \omega_{()}' = \omega_{()} \left[1 - \beta_{()}^2 \right]^{1/2}$$

$$\beta_{()}' = \beta_{()} + \frac{2}{t_D\ \omega_{()}}$$

Appendix D tabulates numerical values of C_{jk} for the DSC Method as a function of frequency, frequency ratio, and strong motion duration time for constant modal damping of 1%, 2%, 5% and 10%. The effect of t_D is most significant at 1% damping and low frequency. For 5% and 10% damping, $t_D = 10$ sec. and 1000 sec. produced similar values for C_{jk} regardless of frequency. The most significant result is that C_{jk} is highly dependent on the damping ratio. For 2%, 5% and 10% damping, $C_{jk} \approx 0.2$, 0.5 and 0.8 respectively, at a frequency ratio of 0.9 (modal frequencies within 10%). In comparison, the definition of closely-spaced modes used in the NRC Grouping and Ten Percent Methods are not damping-dependent.

2.1.5 NRC Double Sum Combination (NRC-DSC)

The NRC-DSC Method (Reference 1) is an adaptation of Rosenblueth's method, described in Section 2.1.4. The coefficients C_{jk} are defined by Equation 2-7. A conservative modification, consistent with the NRC Grouping and Ten Percent methods, is that the product $C_{jk}\,Rp_j\,Rp_k$ is always taken as positive. In Rosenblueth's method, the product may be either positive or negative, depending on the signs of Rp_j and Rp_k. Consequently, NRC-DSC will always produce results ≥ Rosenblueth's method.

2.1.6 Der Kiureghian's Complete Quadratic Combination (CQC)

Der Kiureghian (Reference 13) presents a methodology similar to Rosenblueth's Double Sum Combination (Reference 12) for evaluation of modal correlation for seismic response spectrum analysis. It is also based on application of random vibration theory, but utilizes an infinite duration of white noise to represent seismic loading. The following formula for calculation of the coefficients C_{jk} as a function of modal circular frequencies and modal damping ratios was derived:

$$C_{jk} = \frac{8\,(\beta_j\beta_k\omega_j\omega_k)^{1/2} * (\beta_j\omega_j + \beta_k\omega_k) * \omega_j\omega_k}{(\omega_j^2 - \omega_k^2)^2 + 4\beta_j\beta_k\omega_j\omega_k(\omega_j^2 + \omega_k^2) + 4(\beta_j^2 + \beta_k^2)\omega_j^2\omega_k^2} \qquad \textbf{(Eqn. 2 - 8)}$$

While the form of Equation 2-8 differs significantly from Equation 2-7, the two equations produce equivalent results if t_D is assumed very large in Equation 2-7. This is shown in Appendix D, where C_{jk} is tabulated for DSC with t_D = 1000 sec. and for CQC.

2.1.7 ASCE Standard 4 Recommended Methods

For combination of out-of-phase modal response components, ASCE Standard 4 (Reference 9) specifies the DSC Method (Equation 3200-16). The NRC methods and CQC are also recognized in the commentary.

Draft ASCE Standard 4 (Reference 10) specifies a modified DSC Method (Equation 3200-19) or the CQC Method (Equation 3200-22) as an alternative. The commentary to the Draft ASCE Standard (Reference 10) indicates that Equation 3200-19 produces correlation coefficients "which are practically the same" as Equation 3200-22. Although not indicated, the modified DSC Method presented in Equation 3200-19 was developed by Gupta (Reference 4).

Because they essentially duplicate the DSC and CQC methods, the ASCE methods will not be referenced in the numerical evaluation presented in Chapter 3. However, the numerical results for DSC and CQC are applicable.

2.2 Separation of Modal Responses into Out-of-Phase Components and In-Phase Components

Three methods have received considerable prior review and evaluation: Lindley-Yow (Reference 14), Hadjian (Reference 15), and Gupta (Reference 4). It should be noted that the mathematical statement of each method is not restricted to the mid-frequency range ($f_{SP} < f < f_{ZPA}$) of the response spectrum. However, as discussed at the beginning of this chapter, it is in the mid-frequency range that the separation of individual peak modal responses into out-of-phase and in-phase modal response components is applicable. For $f \leq f_{SP}$, modal responses should be considered out-of-phase and combined by the methods presented in Section 2.1. For $f \geq f_{ZPA}$, modal responses are in-phase and are most accurately accounted for by the method of Kennedy (Reference 3).

The similarities and differences, as well as the limitations, of the three methods are described in the following sections.

2.2.1 Lindley-Yow Method

In its most general form, the Lindley-Yow method (Reference 14) may be defined by the following equations:

$$\alpha_i = ZPA/Sa_i \qquad 0 \leq \alpha_i \leq 1.0 \qquad \text{(Eqn. 2 - 9)}$$

$$Rr_i = R_i * \alpha_i \qquad \text{(Eqn. 2 - 10)}$$

$$Rp_i = R_i * \sqrt{1 - \alpha_i^2} \qquad \text{(Eqn. 2 - 11)}$$

$$Rr = \sum_{i=1}^{n} Rr_i \qquad \text{(Eqn. 2 - 12)}$$

$$Rp = \left[\sum_{j=1}^{n} \sum_{k=1}^{n} C_{jk}\, Rp_j\, Rp_k \right]^{1/2} \qquad \text{(Eqn. 2 - 13)}$$

$$Rt = \sqrt{Rr^2 + Rp^2} \qquad \text{(Eqn. 2 - 14)}$$

where the C_{jk}'s are defined by one of the methods for combining the out-of-phase modal response components described in Section 2.1.

From these mathematical relationships, the following characteristics of the Lindley-Yow method are observed:

- $\alpha_i \rightarrow 1.0$ as $f_i \rightarrow f_{ZPA}$ ($Sa_i = ZPA$). Consequently, $f_{IP} = f_{ZPA}$ in the Lindley-Yow method.

- The in-phase component of modal response for every mode has an associated acceleration equal to the ZPA.

- The out-of-phase component of an individual peak modal response has an associated modified spectral acceleration given by

$$\bar{S}a_i = \left[Sa_i^2 - ZPA^2 \right]^{1/2} \qquad \textbf{(Eqn. 2 - 15)}$$

- $R_i = (Rp_i^2 + Rr_i^2)^{1/2}$; which infers that the in-phase and out-of-phase response components of an individual peak modal response are uncorrelated and, therefore, combine by SRSS.

- All in-phase modal response components (Rr_i) are summed algebraically to obtain Rr.

- All-out-of-phase modal response components (Rp_i) are combined by a suitable method (as described in Section 2.1) to obtain Rp.

- The total response, Rt, is obtained by SRSS combination of Rr and Rp; i.e., Rr and Rp are uncorrelated.

- α_i attains its minimum value at $f_i = f_{SP}$, but increases for $f_i < f_{SP}$ until it attains a value of 1.0 when $Sa_i = ZPA$ in the low frequency region of the spectrum. Values of $\alpha_i > 1.0$ have no meaning because $(1 - \alpha_i^2)^{1/2}$ becomes imaginary.

An obvious limitation of the Lindley-Yow method is in the low frequency range ($f < f_{SP}$) of the response spectrum. There is no physical basis for assuming that low frequency modal responses become increasingly in-phase with the input acceleration time-history, which is an outcome if the Lindley-Yow method is applied to low frequency modal responses. Modal responses in the low frequency range are generally out-of-phase with the input acceleration time history. Therefore, the Lindley-Yow method is applicable to structural systems which do not have significant modal responses with $f_i < f_{SP}$. Lindley and Yow (Reference 14) do not address this limitation. For the sample problems presented in Reference 14, the lowest system frequency is greater than f_{SP} of the applied response spectrum. Therefore, the results reported in Reference 14 are not affected by this limitation. Circumventing this limitation in the Lindley-Yow method is straightforward: apply it only to those modes with $f_i \geq f_{SP}$ and set $\alpha_i = 0$ for $f_i < f_{SP}$.

Professor A.S. Veletsos, one of the project peer reviewers, provided an independent evaluation of the Lindley-Yow method and its limitation, as part of his review of the project results and conclusions. His evaluation is included as Appendix G to this report.

For a structural system with fundamental frequency $\geq f_{SP}$, the Lindley-Yow method lends itself to a relatively straightforward physical interpretation. In the limit, if all modes are retained in the solution, the total mass participation is unity. Applying the Lindley-Yow method is equivalent to performing a static analysis of the system loaded by total mass times the ZPA, and performing the response spectrum analysis for amplified modes $f < f_{ZPA}$ using modified spectral accelerations, $\overline{S}a_i$ given by Equation 2-15. The total dynamic response is then obtained by SRSS combination.

The Lindley-Yow method automatically provides for algebraic combination of modal responses above f_{ZPA}, because $\alpha_i = 1.0$, $Rp_i = 0$, and $Rr_i = R_i$. However, to completely account for the modal response above f_{ZPA}, all system modes of vibration need to be included in the analysis. This contribution is most accurately and efficiently calculated by use of the missing mass method discussed in Section 2.3. Therefore, while in theory, the Lindley-Yow method includes the in-phase contribution from modes above f_{ZPA}, its practical application is for modal responses below f_{ZPA}, coupled with the missing mass method for modal contributions above f_{ZPA}. It is noted that the Lindley-Yow/missing mass approach will produce identical results for any modal analysis cutoff frequency $\geq f_{ZPA}$.

2.2.2 Hadjian Method

The Hadjian Method (Reference 15) is similar in formulation to the Lindley-Yow method, with two notable differences:

- Equation 2-11 is replaced by

$$Rp_i = R_i * (1 - \alpha_i) \quad \textbf{(Eqn. 2 - 16)}$$

- Equation 2-14 is replaced by

$$Rt = |Rp| + |Rr| \quad \textbf{(Eqn. 2 - 17)}$$

- The modified spectral acceleration is given by

$$\overline{S}a_i = Sa_i - ZPA \quad \textbf{(Eqn. 2 - 18)}$$

The Hadjian method has the same limitation as the Lindley-Yow method in the low frequency range, because the definition of α_i is identical. However, the Hadjian Method possesses internal contradictions with respect to the assumed phase relationships between in-phase and out-of-phase response components. Combining Equations 2-10 and 2-16 yields

$$R_i = Rp_i + Rr_i \qquad \textbf{(Eqn. 2 - 19)}$$

This implies that the in-phase and out-of-phase response components for each mode are in-phase with each other. However, all Rr_i's are in-phase and summed algebraically, per Equation 2-12, to obtain Rr. Therefore, it would follow that all Rp_i's are also in-phase and should be summed algebraically to obtain Rp. This contradicts Equation 2-13, in which the Rp_i's are assumed to be predominantly out-of-phase. Kennedy (Reference 3) previously identified this contradiction. On this basis, the Hadjian method is not recommended and is not included in the numerical studies presented in Chapter 3.

2.2.3 Gupta Method

The Gupta Method (Reference 4) is identical in form to the Lindley-Yow method. The one very significant difference is the definition of α_i. Equations 2-10 through 2-14 remain the same. In the Gupta method, α_i is an explicit function of frequency. The original basis for definition of α_i is semi-empirical, derived from numerical studies using actual ground motion records. A best fit equation, which defines α_i as a continuous function of frequency, was developed from the results of the numerical studies.

Two spectrum-dependent frequencies (f_1, f_2) are first defined as follows:

$$f_1 = \frac{Sa_{max}}{2\pi\ Sv_{max}} \qquad \textbf{(Eqn. 2-20)}$$

where Sa_{max} and Sv_{max} are the maximum spectral acceleration and velocity, respectively.

$$f_2 = \left(f_1 + 2\, f_{ZPA}\right)/3 \qquad \textbf{(Eqn. 2-21)}$$

Gupta's definition of α_i is given by:

$$\alpha_i = 0 \text{ for } f_i \le f_1$$

$$\alpha_i = \frac{\ln\left(f_i/f_1\right)}{\ln\left(f_2/f_1\right)} \text{ for } f_1 \le f_i \le f_2 \qquad \textbf{(Eqn. 2-22)}$$

$$\alpha_i = 1.0 \text{ for } f_i \ge f_2$$

For a sharply peaked, in-structure response spectrum,

$$f_1 = f_{SP}$$

because $Sv_{max} = \text{Max}\,(Sa_i / \omega_i) = Sa_{max} / \omega_{SP}$

Substitution into Equation 2-20 yields

$$f_1 = \frac{\omega_{SP}}{2\pi} = f_{SP}$$

The corresponding definition of f_2 yields

$$f_2 = (f_{SP} + 2\,f_{ZPA}) / 3$$

For a sharply peaked, in-structure response spectrum, the Gupta method has the following characteristics:

- For $f_i \le f_{SP}$, $\alpha_i = 0$.
  Consequently, all modal responses with $f_i \le f_{SP}$ are treated as out-of-phase. The limitation in the Lindley-Yow definition of α_i for $f_i \le f_{SP}$ does not apply to Gupta's method.

- For $f_2 \le f_i \le f_{ZPA}$, $\alpha_i = 1.0$
  Consequently, all modal responses with $f_i \ge f_2$ are treated as in-phase. This infers that $f_{IP} = f_2$ in the Gupta method.

- Only modal responses with $f_{SP} < f_i < f_2$ are separated into out-of-phase and in-phase response components.

The potential limitations of the Gupta method lie in the semi-empirical basis for definition of α_i as a function of f_i. The range of applicability is difficult to assess without a comprehensive numerical study using ground and in-structure acceleration records. In Reference 4, Gupta indicates that α_i can be numerically evaluated if the time history used to generate the response spectrum is known. It is implied without stating that numerical evaluation of α_i is more accurate than the semi-empirical definition of α_i given by Equation 2-22.

The overall structure of the Gupta method is superior to the Lindley-Yow method because there is no limitation for modal responses with $f_i < f_{SP}$. In addition, any value of $f_{IP} \leq f_{ZPA}$ can be accommodated by setting $f_2 = f_{IP}$, in lieu of Equation 2-21.

For the initial numerical studies described in Chapter 3, the Lindley-Yow method was selected to evaluate the importance of separating modal responses into out-of-phase and in-phase response components. For the follow-up numerical studies described in Chapter 4, the Gupta method was selected in order to evaluate the influence of f_{IP} on the response spectrum solution. This was accomplished by selecting three different numerical values for f_2.

2.2.4 ASCE Standard 4 Recommended Methods

For separation of in-phase and out-of-phase response components, ASCE Standard 4 (Reference 9) recognizes the Lindley-Yow, Hadjian, and Gupta Methods in the commentary.

Draft ASCE Standard 4 (Reference 10) specifies separation of the in-phase and out-of-phase response components consistent with Gupta's method (Eqns. 3200-18, 3200-20, and 3200-21) except that $f_2 = f_r$ (defined as the "cutoff frequency or ZPA frequency") is substituted for Equation 2-21. The frequency f_r is not clearly defined, but is $\leq f_{ZPA}$. The Lindley-Yow and Hadjian methods are recognized in the commentary to Reference 10.

2.3 Contribution of High Frequency Modes

2.3.1 Missing Mass Method

The "Missing Mass" Method is a convenient, computationally efficient and accurate method to (1) account for the contribution of all modes with frequencies above the frequency (f_{ZPA}) at which the response spectrum returns to the Zero Period Acceleration (ZPA) and (2) account for the contribution to support reactions of mass which is apportioned to system support points. It constitutes the total effect of all system mass which does not participate in (i.e., "missing" from) the modes with frequencies below f_{ZPA}. The system response to the missing mass is calculated by performing a static analysis for applied loads equal to the missing mass multiplied by the spectrum ZPA. This method is mathematically rigorous and is considered the only acceptable method to account for high frequency modal contributions ($f \geq f_{ZPA}$) and mass apportioned to system support points.

Kennedy (Reference 3) documented this method and recommended that it be included in Regulatory Guidance. The 1989 revision to the SRP Section 3.7.2, "Seismic System Analysis," (Reference 2) incorporated Kennedy's recommendation as Appendix A. The mathematical details are presented in both References 2 and 3. For completeness, the mathematical formulation is included as Appendix I to this report. The guideline provided in References 2 and 3, that the missing mass contribution needs to be considered only if the fraction of missing mass at any degree of freedom exceeds 0.1, should be eliminated. This guideline does not consider the total mass which is missing, which in the limit could be 10%. In a static analysis this represents a 10% reduction in the applied load. The missing mass contribution should be calculated in all response spectrum analyses, because its potential effect on support reactions is difficult to judge based on the fraction of missing mass. This calculation has been automated in a number of piping analysis codes and does not represent a significant computational effort.

The missing mass contribution to the response spectrum analysis solution represents response which is completely in-phase with the time varying acceleration input and can be scaled to the instantaneous acceleration to obtain its contribution at any specific point in time. This characteristic is not important in response spectrum analysis because only peak response is predicted. In this case, the ZPA is used to generate the missing mass loading. However, the importance of the missing mass contribution is not limited to response spectrum analysis only. Mode superposition time history analysis is most accurately and efficiently performed by a procedure similar to that employed in response spectrum analysis (Reference 4). Only modes which vibrate at frequencies below f_{ZPA} need to be included in the transient mode superposition solution. The missing mass contribution, scaled to the instantaneous acceleration, is then algebraically summed with the transient solution at the corresponding time to obtain the total solution. This method is more rigorous and accurate than including additional modes in the transient mode superposition solution. Even if additional modes are included, it is still necessary to calculate the missing mass for the excluded, higher frequency modes and system support points. This is quantitatively demonstrated in Appendix E of this report.

Use of the Missing Mass method for calculating the contribution of high frequency modes is recommended in Draft ASCE Standard 4 (Reference 10) for both response spectrum analysis (Eqn. 3200-8) and mode superposition time history analysis (Eqn. 3200-5). In Reference 10, this is referred to as the "residual rigid response due to the missing mass."

2.3.2 Static ZPA Method

The Lindley-Yow Method (Reference 14) defines the acceleration of the in-phase response component of all modes to be the ZPA of the response spectrum. As discussed in Section 2.2.1, the algebraic summation of the in-phase response components for all modes (Rr) is equivalent to the static response for a load equal to the total mass times the ZPA. When using the Lindley-Yow method, an alternate approach to including the contribution of high frequency ($f \geq f_{ZPA}$) modes is to calculate Rr directly by the Static ZPA method. This eliminates the need for calculation of the missing mass, since it is automatically included in the static analysis of total mass times ZPA. The out-of-phase response component (Rp) is calculated in accordance with the Lindley-Yow method.

During the course of numerically verifying the equivalence between Rr calculated by Lindley-Yow plus Missing Mass approach and Rr calculated by the Static ZPA Method, a significant result was obtained which led to a supplementary study of differences in mass distribution defined in a dynamic analysis model and in a static analysis model. Using a BNL version of SAP V adapted for piping analysis, correlation was not initially achieved. Further investigation identified the source of the discrepancy to be the different treatments of the piping system mass. Calculation of Rr by the Lindley-Yow plus Missing Mass approach utilized the dynamic analysis model, in which the distributed piping system mass is replaced by discrete masses at the nodes of the model. Calculation of Rr by the Static ZPA Method initially utilized a static analysis model, in which the mass remains distributed along the pipe elements. For the BM3 model, the differences in mass distribution led to significant discrepancies in the support reactions predicted by the two analyses. When the discrete mass distribution from the dynamic analysis model was utilized in the static ZPA method, all discrepancies disappeared and equivalence between the two approaches to calculate Rr was verified. This issue is further evaluated in Appendix C; guidelines for ensuring that the model discretization is sufficient to accurately represent the distributed mass are also provided.

2.4 Complete Solution for Response Spectrum Analysis

For the numerical studies presented in Chapter 3, three approaches are used to define the complete response spectrum analysis solution. For simplicity, these have been designated Method 1, Method 2, and Method 3, and are defined below. The coefficients C_{jk} are defined by one of the out-of-phase combination methods (Section 2.1.1 through 2.1.6). In the numerical studies, all six methods were tested in conjunction with Methods 1, 2, and 3.

2.4.1 Method 1

Method 1 represents the common method applied to response spectrum analysis since the 1980's. Amplified modal responses ($f < f_{ZPA}$) are combined by SRSS with a correction for closely spaced modal frequencies. The contribution of unamplified modal responses ($f > f_{ZPA}$) is calculated by the missing mass method of Section 2.3.1. These two components are then combined by SRSS to produce the total solution. Mathematically, this is represented by

$$Rp = \left[\sum_{j=1}^{n}\sum_{k=1}^{n} C_{jk} R_j R_k\right]^{1/2}$$

$$n = \text{no. of modes below } f_{ZPA} \qquad \text{(Eqn. 2-23)}$$

$$Rr = R_{\text{missing mass}}$$

$$Rt = \sqrt{Rp^2 + Rr^2}$$

2.4.2 Method 2

Method 2 introduces the concept of in-phase and out-of-phase modal response components for the amplified modes ($f < f_{ZPA}$). Mathematically, the complete solution is represented by

$$Rp_i = R_i * (1 - \alpha_i^2)^{1/2}$$

$$Rr_i = R_i * \alpha_i$$

$$Rp = \left[\sum_{j=1}^{n}\sum_{k=1}^{n} C_{jk} Rp_j Rp_k\right]^{1/2}$$

$$n = \text{no. of modes below } f_{ZPA} \qquad \text{(Eqn. 2-24)}$$

$$Rr = \sum_{i=1}^{n} Rr_i + R_{\text{missing mass}}$$

$$Rt = \sqrt{Rp^2 + Rr^2}$$

The method recommended in Draft ASCE Standard 4 (Reference 10) for obtaining the complete response spectrum analysis solution (Eqns. 3200-17 and 3200-18) is essentially equivalent to Method 2.

Method 2 is equally applicable to both the Lindley-Yow (Section 2.2.1) and the Gupta (Section 2.2.3) methods. Only the definition of α_i changes. For the initial numerical study described in

Chapter 3, the Lindley-Yow method was selected for implementation. For the follow-up numerical study described in Chapter 4, the Gupta method was implemented.

2.4.3 Method 3

Method 3 is a variation of Method 2, which utilizes the Static ZPA Method of Section 2.3.2 to calculate Rr. Mathematically, the complete solution is represented by

$$Rp_i = R_i * (1 - \alpha_i^2)^{1/2}$$

$$Rp = \left[\sum_{j=1}^{n} \sum_{k=1}^{n} C_{jk}\, Rp_j\, Rp_k \right]^{1/2}$$

n = no. of modes below f_{ZPA} **(Eqn. 2-25)**

$$Rr = R_{static\ ZPA}$$

$$Rt = \sqrt{Rp^2 + Rr^2}$$

Method 3 is compatible only with the Lindley-Yow method, because calculation of Rr by the Static ZPA Method is based on the Lindley-Yow definition for α_i, per Equation 2-9.

3 QUANTITATIVE STUDIES OF MODAL RESPONSE COMBINATION METHODS

In addition to the qualitative review presented in Section 2, a series of analyses were conducted to provide a quantitative basis for recommending revisions to regulatory guidance on modal combination methods. A piping system model previously utilized in the NUREG/CR-5627 study conducted by BNL (Reference 6) was selected for analysis. This model is designated BM3. Detailed descriptions of the BM3 model and the seismic loading are presented in Appendices A and B respectively. A brief description of the model and loading is provided in Section 3.1

3.1 BM3 Piping Analysis Model

Figure 3-1 shows a schematic of the BM3 piping model, including node, element, support, and pipe size information. Table 3-1 provides a list of the first 31 natural frequencies (up to 70 Hz) for the BM3 model. Appendix A also includes a printout of the computer model, the nodal mass distribution, and the mode participation factors. Figures 3-2 and 3-3 show the 1% and 5% damping, unbroadened acceleration response spectra of the input time history. The 1% response spectrum was used as x-direction input for all response spectrum analysis (RSA) conducted, with one exception. One analysis using the 5% spectrum was conducted in order to assess the effect of damping on the behavior of the modal response combination methods.

The input response spectrum is typical of an in-structure response spectrum which results from filtering and amplification of the seismic ground motion through the building structure/soil system. It exhibits a sharp, highly amplified peak at the fundamental frequency of the structure/soil system. Piping systems attached to building structures are usually subjected to this type of seismic input, instead of ground motion associated with broad-banded response spectra. For this study, the unbroadened spectrum was used to provide a direct comparison to time history analysis results. The input time history of acceleration used to generate the response spectra and to perform the time history analyses is provided in Appendix B.

For the BM3 input response spectra, $f_{ZPA} \approx 16.5$ Hz and the ZPA = 0.54 g. The BM3 model has 14 modes below f_{ZPA}. These modes are associated with the amplified modal responses. From Table 3-1, the following modes are closely spaced, based on a 10% criterion: modes 3 and 4; modes 5 and 6; modes 6 and 7; modes 8, 9, and 10; modes 9, 10 and 11; modes 11 and 12. The number of closely spaced modes was judged to be adequate to assess differences between the six (6) methods tested for combination of the out-of-phase modal responses. For development of complete RSA solutions utilizing Methods 1, 2, and 3 defined in Section 2.4, only the first 14 modes ($f < f_{ZPA}$) are needed.

3.2 Description of Analyses Performed

3.2.1 Baseline Time History Analysis

To provide a baseline for assessing the accuracy of RSA methods, time history analysis of BM3 was conducted using the acceleration input tabulated in Appendix B. Both mode superposition and direct integration solutions were obtained. Appendix E contains a discussion of the comparison between the two time history solutions and the tabulated results for support reactions and pipe end moments, for 1% and 5% damping.

In the NUREG/CR-5627 study (Reference 6), a mode superposition time history analysis had been performed using the first 31 modes of the BM3 model (f_{31} = 70 Hz). In this study, a direct integration analysis was performed and compared to the previously obtained solution. Significant discrepancies for a number of support reactions prompted additional investigation of the time history solutions. It was determined that the missing mass contribution to the mode superposition solution was significant for these support reactions. Inclusion of the missing mass in the mode superposition solution resolved the discrepancies. The tabulated results in Appendix E show the mode superposition solution with and without the missing mass contribution, as well as comparison to the direct integration solution.

The excellent correlation obtained between the mode superposition and the direct integration solutions demonstrates that constant modal damping can be reasonably approximated in the direct integration method by careful selection of the target frequencies for the calculation of the damping coefficients α and β. Guidelines for performing both mode superposition and direct integration analysis are provided in Appendix E.

For comparison to complete RSA results, the mode superposition solution with the missing mass contribution was used as the baseline. There would be negligible differences in the comparison if the direct integration solution was used as the baseline. The selection was made because several comparisons to partial modal summations are included in the quantitative studies; baselines for these partial modal solutions can only be developed by the mode superposition method.

3.2.2 Response Spectrum Analysis

A series of eight (8) different comparisons between time history and RSA solutions were generated. These results are presented in Tables 3-2 through 3-17, for support reactions and pipe end moments. Pipe end moments are defined as $(Mx^2 + My^2 + Mz^2)^{1/2}$. In each Table, predictions using all six of the out-of-phase response combination methods defined in Section 2.1 are presented. Actual numerical values are shown for the baseline time history solution. The RSA results are shown as the ratio of the RSA numerical value to the time history numerical value. This form of presentation was chosen to facilitate easy recognition of overprediction and underprediction by the RSA methods. A ratio of 1.0 indicates exact agreement; a ratio greater than 1.0 indicates RSA overprediction; a ratio less than 1.0 indicates RSA underprediction. The mean value and standard deviation of the ratios are shown at the bottom of the table for each RSA method, as an indication of the overall correlation and scatter, compared to the time history solution. In calculating the mean and standard deviation of the ratios, all output locations were weighted equally, regardless of their response magnitude.

The eight (8) comparative analyses are

1) 8 Mode Time History vs. 8 Mode RSA Combination, $Rp_i = R_i$

2) 14 Mode Time History vs. 14 Mode RSA Combination, $Rp_i = R_i$

3) 31 Mode Time History vs. 31 Mode RSA Combination, $Rp_i = R_i$

4) Complete Time History vs. Method 1, n = 14

5) Complete Time History vs. Method 2 (Lindley-Yow), n=14

6) Complete Time History vs. Method 2 (Lindley-Yow), n=31

7) Complete Time History vs. Method 3, n = 14

8) Complete Time History vs. Method 3, n = 14 (5% damping)

Analysis No.'s 1, 2, and 3 were conducted to study the ability of the six (6) out-of-phase response combination methods to match the time history results as the number of included modes is increased. For these three analyses, the modal responses, R_i, are assumed to be out-of-phase; i.e., $Rp_i = R_i$. This method of RSA was used in the early applications of seismic analysis in the nuclear power industry.

Analysis No.'s 4, 5, and 7 were conducted to evaluate the ability of Methods 1, 2, and 3 to match the complete time history solution. Analysis No. 8 tested the sensitivity of the results to a change in damping from 1% to 5%. Analysis No. 6 was included to verify that Method 2 produces the same results when additional modes above f_{ZPA} are included in the analysis.

The equivalency between Rr calculated by Method 2 and Rr calculated by Method 3 was also verified by a separate numerical study. Table 3-18 compares the results of four (4) procedures to obtain Rr, including the Static ZPA Method. In applications of the Static ZPA Method, the dynamic lumped mass representation is used. See the discussion in Section 2.3.2 and Appendix C.

3.3 Discussion of Numerical Results

3.3.1 Out-of-Phase Modal Response Combination

Tables 3-2 through 3-7 present the 8, 14, and 31 mode RSA comparisons to time history results. The 8 mode RSA shows the best correlation to time history. This is reasonable because the highly amplified responses are predominately out-of-phase with the acceleration input. The 14 mode RSA shows poorer correlation to time history. The six (6) additional modal responses, up to f_{ZPA}, would be expected to have a significant in-phase component, which is not appropriately treated by the out-of-phase modal response combination methods. The 31 mode RSA shows the poorest correlation to time history. The additional 17 modes with $f > f_{ZPA}$ are completely in-phase with each other. The out-of-phase modal combination methods are inappropriate.

Among the combination methods, the RB-DSC and CQC methods consistently show the best correlation to the time history solution. The NRC Grouping, Ten percent and DSC methods produce the most conservative predictions for locations where the time history results are exceeded. However they produce very similar predictions for locations where the time history results are either underpredicted or are reasonably matched. For the BM3 analysis, SRSS performed better overall than the three NRC methods, but worse than the RB-DSC and CQC methods. The increased conservatism introduced by the three NRC methods does not appear to be justified, because there is only a minimal corresponding improvement when RSA underpredicts the time history results.

The small difference in predictions between the RB-DSC and CQC methods is attributed to the definition of t_D=15 seconds for the RB-DSC method. Consequently, the mode correlation coefficients for RB-DSC are somewhat larger than the CQC coefficients. See Appendix D.

3.3.2 Complete Solution

Tables 3-8 through 3-17 present comparisons of RSA Methods 1, 2, and 3 to the complete time history solution. Method 1 represents the common RSA methodology utilized since the 1980's. Method 1 produces consistently inferior results when compared to Methods 2 and 3. This is attributed to the lack of consideration of the in-phase response component for the amplified modal responses. Both overpredictions and underpredictions are more extreme, which is consistent with the treatment of in-phase response as out-of-phase response.

Methods 2 and 3 produce essentially identical results, which are in reasonable agreement with the time history solution. The best correlation with the least scatter is achieved with the RB-DSC and CQC methods.

The largest overpredictions occur at support reactions of small magnitude, relative to the largest support reactions. Differences among the mode combination methods are also most extreme at these support reactions. This behavior is most likely attributed to the absence of a dominant modal contribution to the total support reaction. Without a dominant modal contribution, the limited capacity of response spectrum analysis methods to recreate the actual phase relationships between modal responses is more likely to result in less accurate predictions. A typical example is Node 1, M_x.

Comparison of Tables 3-10 and 3-11 to Tables 3-12 and 3-13 demonstrates the insensitivity of the Lindley-Yow Method (Method 2) to the number of modes included in the RSA, provided all modes below f_{zpa} are included. Table 3-18 provides a comparison of four (4) approaches to calculate the total in-phase response component, Rr. The equivalence between Rr for Method 2 and Rr for Method 3 is evident.

Results for the 5% damping input spectrum are presented in Tables 3-16 and 3-17, utilizing Method 3. The greater accuracy of the RB-DSC and CQC combination methods is more evident at 5% damping than at 1% damping. At 5% damping, the coefficients C_{jk} are numerically larger than at 1% damping (see Appendix D). Consequently, differences between the combination methods are more pronounced. The overall level of conservatism for RSA methods is higher at 5% damping than at 1%; however, the scatter is also significantly larger. The NRC methods and SRSS exhibit the largest increases in conservatism and scatter.

3.4 Summary of Results

Based on the quantitative studies described above, the following observations are made:

- Including modes above f_{zpa} in the out-of-phase modal response combination is always incorrect and cannot be justified. The contribution of modes above f_{zpa} is accurately calculated by the "missing mass" procedure and it is considered mandatory to include this contribution.

- Assuming amplified modal responses ($f<f_{zpa}$) have only out-of-phase components can lead to significant error in the RSA prediction. Method 1, which is based on this assumption, is significantly less accurate than Methods 2 and 3, which incorporate separation of modal responses into in-phase and out-of-phase response components.

- Of the six methods tested for combination of the out-of-phase response components, RB-DSC and CQC consistently produce the most accurate results, and are essentially equivalent to each other. Increased accuracy is most notable in the 5% damping case. SRSS compared fairly well to RB-DSC and CQC for 1% damping, but deviated significantly for 5% damping.

- Method 2, using the Lindley-Yow formulation, and Method 3 are equivalent; they provide two approaches to achieve the same solution. Method 3 involves less computation because the total in-phase response component is calculated in a single step.

Table 3-1
BM3 Model Modal Frequencies

Mode Number	ω (radians/sec)	f (cycles/sec)
1	18.3	2.91
2	27.6	4.39
3	34.7	5.52
4	35.8	5.70
5	43.8	6.98
6	46.1	7.34
7	49.5	7.88
8	64.7	10.30
9	69.5	11.06
10	70.6	11.23
11	72.2	11.50
12	78.1	12.43
13	87.2	13.88
14	101.3	16.12
15	113.3	18.04
16	117.0	18.62
17	122.7	19.52
18	122.9	19.56
19	137.1	21.82
20	139.4	22.18
21	143.6	22.86
22	163.1	25.95
23	245.2	39.02
24	248.3	39.52
25	258.9	41.21
26	288.5	45.91
27	296.9	47.25
28	330.7	52.64
29	372.5	59.29
30	440.4	70.09
31	444.2	70.70

Table 3-2
BM3 Model
8 Mode Combination
Time History vs. Response Spectrum
Support Reactions

Forces in lbs
Moments in in-lbs

1 % Damping

NODE NO.	REAC. TYPE	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	Fx	3.34	0.87	0.87	0.88	0.88	0.87	0.87
1	Fy	2.91	1.07	1.07	1.08	1.08	1.07	1.07
1	Fz	0.93	1.50	1.53	1.54	1.53	1.49	1.50
1	Mx	32.25	0.83	0.91	0.93	0.87	0.81	0.81
1	My	724.63	0.91	0.91	0.91	0.91	0.91	0.91
1	Mz	187.36	0.90	0.90	0.91	0.91	0.90	0.90
4	Fx	25.04	0.88	0.88	0.88	0.88	0.88	0.88
4	Fz	11.26	1.44	1.46	1.47	1.46	1.43	1.43
7	Fy	13.30	1.12	1.13	1.19	1.14	1.11	1.12
11	Fy	13.43	1.37	1.40	1.64	1.42	1.34	1.35
11	Fz	70.88	0.85	0.85	0.85	0.85	0.85	0.85
15	Fx	484.98	0.75	0.76	0.76	0.76	0.76	0.75
17	Fy	25.01	1.90	1.92	2.32	1.97	1.85	1.87
17	Fz	36.05	1.52	1.53	1.75	1.56	1.55	1.54
36	Fy	81.70	1.00	1.01	1.02	1.01	1.00	1.00
36	Fz	36.61	1.35	1.51	1.78	1.44	1.43	1.40
38	Fx	41.85	1.19	1.21	1.23	1.20	1.18	1.19
38	Fy	49.23	1.02	1.03	1.03	1.03	1.02	1.02
38	Fz	12.26	1.39	1.39	1.40	1.39	1.39	1.39
38	Mx	602.71	2.14	2.22	2.29	2.18	2.15	2.14
38	My	883.44	1.33	1.33	1.34	1.33	1.32	1.33
38	Mz	3486.10	1.02	1.03	1.03	1.03	1.02	1.02
23	Fx	137.41	1.16	1.16	1.16	1.16	1.16	1.16
23	Fy	69.96	1.10	1.14	1.23	1.13	1.10	1.10
31	Fx	6.33	1.00	1.21	1.23	1.05	0.95	0.97
31	Fy	16.51	1.17	1.25	1.28	1.20	1.15	1.16
31	Fz	18.29	1.66	1.95	1.97	1.74	1.58	1.61
31	Mx	1486.40	1.58	1.94	1.94	1.67	1.48	1.52
31	My	129.92	1.45	1.76	1.81	1.55	1.37	1.41
31	Mz	446.70	1.22	1.53	1.56	1.31	1.14	1.17
Mean Value of 30 Comp.			1.22	1.29	1.35	1.25	1.21	1.21
Standard Dev. of 30 Comp.			0.33	0.38	0.43	0.35	0.32	0.33

Table 3-3
BM3 Model
8 Mode Combination
Time History vs. Response Spectrum
Pipe End Moments

Moments in in-lbs $(M_x^2+M_y^2+M_z^2)^{1/2}$ 1 % Damping

ELEM. NO.	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	727.97	0.90	0.90	0.90	0.90	0.90	0.90
2	720.75	0.89	0.89	0.89	0.89	0.89	0.89
3	851.89	0.91	0.91	0.92	0.92	0.91	0.91
4	718.23	0.98	0.98	0.98	0.98	0.98	0.98
5	631.26	1.03	1.03	1.04	1.03	1.03	1.03
6	1999.70	0.95	0.96	0.96	0.96	0.95	0.95
7	1826.20	0.96	0.96	0.96	0.96	0.96	0.96
8	1049.10	1.01	1.02	1.06	1.02	1.00	1.00
9	1769.90	1.01	1.01	1.01	1.01	1.01	1.01
10	1675.30	1.00	1.01	1.03	1.01	1.00	1.00
11	3931.90	0.86	0.86	0.86	0.86	0.86	0.86
12	7628.70	0.87	0.87	0.88	0.87	0.87	0.87
13	6299.40	0.90	0.90	0.91	0.90	0.90	0.90
14	9414.80	0.71	0.71	0.73	0.71	0.71	0.71
15	4056.20	0.75	0.75	0.81	0.76	0.76	0.75
16	3257.00	0.77	0.78	0.86	0.79	0.78	0.78
17	2532.00	1.72	1.73	2.10	1.78	1.71	1.72
18	2696.00	1.03	1.04	1.11	1.05	1.03	1.03
19	1937.60	1.16	1.17	1.31	1.19	1.16	1.16
20	7915.90	1.12	1.13	1.15	1.13	1.13	1.13
21	1836.00	1.23	1.26	1.27	1.24	1.22	1.22
22	2268.30	1.07	1.10	1.10	1.08	1.06	1.07
23	2151.00	1.08	1.10	1.11	1.09	1.07	1.07
24	1751.20	1.10	1.13	1.14	1.11	1.09	1.09
25	538.77	1.17	1.45	1.48	1.25	1.09	1.12
26	495.58	1.26	1.53	1.57	1.34	1.19	1.22
27	327.90	1.97	2.20	2.26	2.04	1.91	1.94
28	440.31	1.86	1.92	1.95	1.89	1.85	1.85
29	456.61	1.75	1.78	1.80	1.77	1.75	1.75
30	1522.40	1.58	1.95	1.95	1.68	1.49	1.53
31	8396.90	1.19	1.21	1.22	1.20	1.19	1.19
32	8240.10	1.21	1.23	1.24	1.22	1.21	1.21
33	7816.90	1.21	1.22	1.24	1.21	1.21	1.21
34	6420.40	1.08	1.09	1.09	1.08	1.08	1.08
35	6909.70	1.06	1.06	1.07	1.06	1.06	1.06
36	1887.80	1.26	1.27	1.29	1.27	1.26	1.26
37	3598.70	1.10	1.11	1.12	1.11	1.10	1.10
Mean Value of 37 Comp.		1.13	1.17	1.20	1.14	1.12	1.12
Standard Dev. of 37 Comp.		0.30	0.35	0.37	0.32	0.29	0.29

Table 3-4
BM3 Model
14 Mode Combination
Time History vs. Response Spectrum
Support Reactions

Forces in lbs
Moments in in-lbs

1 % Damping

NODE NO.	REAC. TYPE	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	Fx	3.96	0.79	0.83	0.90	0.84	0.79	0.79
1	Fy	3.51	1.55	1.65	2.11	1.75	1.41	1.44
1	Fz	2.57	2.91	3.00	4.10	3.29	2.51	2.63
1	Mx	43.23	6.37	6.57	9.05	7.22	5.45	5.71
1	My	794.03	0.87	0.87	0.91	0.88	0.86	0.86
1	Mz	215.80	0.97	1.01	1.19	1.05	0.93	0.94
4	Fx	29.24	0.83	0.84	0.92	0.86	0.81	0.82
4	Fz	27.64	2.72	2.80	3.83	3.07	2.34	2.45
7	Fy	13.87	1.12	1.20	1.28	1.18	1.10	1.10
11	Fy	13.25	1.48	1.55	1.85	1.60	1.43	1.44
11	Fz	93.01	0.86	0.87	1.00	0.91	0.82	0.83
15	Fx	706.21	0.62	0.63	0.65	0.63	0.63	0.62
17	Fy	25.56	1.91	1.95	2.36	1.99	1.85	1.88
17	Fz	62.29	1.27	1.37	1.48	1.31	1.28	1.28
36	Fy	46.89	1.93	2.16	2.27	2.03	1.91	1.92
36	Fz	68.17	1.15	1.37	1.47	1.21	1.16	1.16
38	Fx	121.91	0.98	1.09	1.20	1.02	1.02	1.00
38	Fy	43.98	1.20	1.22	1.25	1.21	1.20	1.20
38	Fz	42.65	0.90	1.05	1.14	0.96	0.96	0.94
38	Mx	713.03	1.97	2.27	2.41	2.09	1.95	1.96
38	My	2783.00	0.93	1.07	1.16	0.99	0.98	0.97
38	Mz	3117.80	1.19	1.21	1.24	1.20	1.19	1.19
23	Fx	159.21	1.35	1.43	1.52	1.39	1.39	1.38
23	Fy	28.00	3.76	4.72	5.18	4.12	3.68	3.72
31	Fx	6.99	1.45	1.70	1.85	1.56	1.46	1.46
31	Fy	13.95	1.78	1.96	2.05	1.85	1.80	1.79
31	Fz	15.50	2.04	2.43	2.46	2.15	1.95	1.98
31	Mx	1102.30	2.21	2.72	2.75	2.35	2.09	2.14
31	My	259.45	1.09	1.32	1.41	1.17	1.11	1.11
31	Mz	608.00	1.31	1.59	1.70	1.41	1.31	1.32
Mean Value of 30 Comp.			1.65	1.82	2.09	1.78	1.58	1.60
Standard Dev. of 30 Comp.			1.14	1.24	1.67	1.30	0.97	1.02

Table 3-5
BM3 Model
14 Mode Combination
Time History vs. Response Spectrum
Pipe End Moments

Moments in in-lbs $(M_x^2+M_y^2+M_z^2)^{1/2}$ 1 % Damping

ELEM. NO.	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES)					
		SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	776.55	0.92	0.93	1.00	0.95	0.90	0.91
2	759.36	0.91	0.91	0.97	0.93	0.89	0.90
3	968.28	1.22	1.25	1.55	1.32	1.12	1.15
4	773.63	1.06	1.07	1.20	1.10	1.02	1.03
5	701.53	1.36	1.39	1.71	1.47	1.26	1.29
6	2207.60	0.93	0.94	0.99	0.95	0.91	0.92
7	1948.20	0.94	0.94	0.98	0.95	0.93	0.93
8	837.44	2.75	2.86	3.77	3.08	2.43	2.52
9	1357.40	2.12	2.18	2.73	2.31	1.93	1.99
10	2183.80	0.96	0.96	1.09	1.00	0.92	0.93
11	4173.00	0.82	0.83	0.84	0.83	0.82	0.82
12	8283.10	0.81	0.81	0.82	0.82	0.81	0.81
13	6579.90	0.87	0.87	0.88	0.87	0.87	0.87
14	13174.00	0.59	0.60	0.63	0.61	0.60	0.60
15	2294.00	1.50	1.55	1.72	1.55	1.54	1.53
16	1693.10	1.99	2.09	2.37	2.09	2.06	2.04
17	2456.50	1.84	1.91	2.29	1.92	1.84	1.84
18	2377.20	1.23	1.26	1.35	1.25	1.22	1.22
19	1635.50	1.45	1.49	1.67	1.49	1.44	1.45
20	7586.00	1.25	1.32	1.36	1.27	1.25	1.25
21	1616.20	1.49	1.61	1.66	1.54	1.49	1.49
22	1977.10	1.51	1.60	1.70	1.56	1.54	1.53
23	1900.20	1.51	1.61	1.70	1.56	1.54	1.53
24	1607.70	1.52	1.64	1.73	1.57	1.55	1.54
25	265.36	3.01	3.74	4.00	3.29	2.89	2.94
26	271.57	2.87	3.53	3.79	3.13	2.78	2.82
27	348.19	2.11	2.49	2.63	2.26	2.07	2.09
28	552.58	1.65	1.75	1.79	1.69	1.65	1.65
29	581.00	1.56	1.64	1.67	1.59	1.56	1.56
30	1102.30	2.34	2.87	2.92	2.49	2.22	2.27
31	7586.80	1.61	1.75	1.84	1.67	1.65	1.64
32	9093.40	1.41	1.54	1.64	1.46	1.44	1.43
33	11281.00	1.15	1.28	1.36	1.20	1.18	1.17
34	6061.80	1.21	1.22	1.24	1.22	1.21	1.21
35	6634.10	1.22	1.28	1.32	1.24	1.23	1.23
36	2451.60	1.18	1.30	1.37	1.22	1.20	1.19
37	4009.70	1.18	1.27	1.33	1.21	1.20	1.19
Mean Value of 37 Comp.		1.46	1.58	1.72	1.53	1.44	1.45
Standard Dev. of 37 Comp.		0.58	0.73	0.84	0.65	0.55	0.56

Table 3-6
BM3 Model
31 Mode Combination
Time History vs. Response Spectrum
Support Reactions

Forces in lbs
Moments in in-lbs

1 % Damping

NODE NO.	REAC. TYPE	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	Fx	8.40	0.82	0.83	0.85	0.83	0.82	0.82
1	Fy	10.14	2.28	2.34	2.39	2.34	2.31	2.31
1	Fz	1.60	4.82	5.01	6.75	5.43	4.19	4.37
1	Mx	49.98	5.63	5.85	7.95	6.36	4.85	5.07
1	My	776.73	0.89	0.89	0.93	0.90	0.88	0.88
1	Mz	193.46	2.72	2.79	2.87	2.80	2.73	2.74
4	Fx	68.75	0.66	0.67	0.68	0.67	0.66	0.66
4	Fz	19.55	3.92	4.07	5.49	4.42	3.40	3.55
7	Fy	13.31	1.19	1.28	1.36	1.25	1.17	1.17
11	Fy	13.31	1.51	1.60	1.88	1.64	1.43	1.44
11	Fz	81.14	1.00	1.00	1.15	1.04	0.95	0.96
15	Fx	713.32	0.61	0.62	0.64	0.62	0.62	0.62
17	Fy	25.61	1.90	1.95	2.35	1.98	1.85	1.87
17	Fz	64.26	1.24	1.33	1.45	1.28	1.25	1.25
36	Fy	46.31	1.96	2.19	2.30	2.06	1.94	1.95
36	Fz	48.85	2.07	2.33	2.44	2.15	2.08	2.08
38	Fx	128.79	0.95	1.06	1.16	1.00	1.00	0.98
38	Fy	43.52	1.21	1.24	1.26	1.23	1.21	1.21
38	Fz	31.40	1.26	1.47	1.58	1.34	1.33	1.31
38	Mx	720.04	1.95	2.25	2.39	2.07	1.93	1.94
38	My	2142.00	1.23	1.42	1.54	1.30	1.30	1.28
38	Mz	3088.90	1.20	1.22	1.25	1.22	1.20	1.20
23	Fx	259.22	0.88	0.93	0.99	0.91	0.91	0.90
23	Fy	26.04	4.07	5.11	5.60	4.47	3.98	4.01
31	Fx	23.24	0.84	0.88	0.91	0.86	0.84	0.84
31	Fy	14.21	1.75	1.92	2.02	1.82	1.77	1.76
31	Fz	16.07	1.97	2.34	2.38	2.08	1.88	1.92
31	Mx	1136.90	2.14	2.64	2.67	2.28	2.02	2.07
31	My	608.43	0.83	0.89	0.91	0.85	0.83	0.83
31	Mz	1767.90	0.81	0.87	0.89	0.83	0.81	0.81
Mean Valur of 30 Comp.			1.81	1.97	2.23	1.93	1.74	1.76
Standard Dev. of 30 Comp.			1.27	1.38	1.84	1.44	1.10	1.15

Table 3-7
BM3 Model
31 Mode Combination
Time History vs. Response Spectrum
Pipe End Moments

Moments in in-lbs $(M_x^2+M_y^2+M_z^2)^{1/2}$ 1 % Damping

ELEM. NO.	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	823.39	0.91	0.92	0.98	0.93	0.89	0.90
2	818.58	0.87	0.87	0.92	0.88	0.85	0.86
3	1412.20	0.96	0.98	1.16	1.02	0.90	0.92
4	762.56	1.08	1.09	1.23	1.13	1.04	1.05
5	598.91	1.66	1.69	2.05	1.78	1.54	1.57
6	2134.40	0.97	0.97	1.02	0.99	0.95	0.96
7	1876.40	0.98	0.98	1.01	0.99	0.97	0.97
8	852.96	2.70	2.82	3.71	3.03	2.39	2.48
9	1415.50	2.03	2.09	2.62	2.22	1.86	1.91
10	1945.30	1.07	1.08	1.23	1.12	1.03	1.04
11	3826.10	0.90	0.91	0.92	0.91	0.90	0.90
12	8440.50	0.80	0.80	0.81	0.80	0.80	0.80
13	6741.60	0.85	0.85	0.86	0.85	0.85	0.85
14	13241.00	0.59	0.60	0.63	0.60	0.60	0.60
15	2290.80	1.50	1.55	1.72	1.55	1.54	1.53
16	1698.70	1.99	2.09	2.36	2.08	2.05	2.03
17	2455.40	1.84	1.91	2.29	1.92	1.84	1.84
18	2360.10	1.24	1.27	1.36	1.26	1.23	1.23
19	1623.90	1.46	1.50	1.68	1.50	1.46	1.46
20	7539.20	1.26	1.33	1.36	1.28	1.26	1.26
21	1595.60	1.53	1.65	1.70	1.57	1.53	1.53
22	2042.80	1.47	1.56	1.65	1.52	1.50	1.49
23	1800.80	1.61	1.71	1.81	1.66	1.63	1.63
24	1522.10	1.66	1.78	1.89	1.72	1.69	1.68
25	926.28	1.23	1.38	1.44	1.29	1.20	1.21
26	665.57	1.55	1.76	1.85	1.63	1.52	1.53
27	399.00	1.97	2.28	2.40	2.09	1.93	1.94
28	742.38	1.34	1.40	1.44	1.37	1.34	1.34
29	824.73	1.26	1.32	1.34	1.29	1.27	1.27
30	1989.00	1.44	1.71	1.73	1.52	1.38	1.40
31	7437.70	1.65	1.79	1.88	1.70	1.68	1.67
32	8931.50	1.43	1.57	1.67	1.49	1.47	1.46
33	11421.00	1.14	1.27	1.35	1.19	1.17	1.16
34	6096.60	1.20	1.21	1.24	1.21	1.21	1.21
35	6633.90	1.22	1.28	1.32	1.24	1.23	1.23
36	1942.80	1.51	1.67	1.76	1.57	1.54	1.53
37	3538.70	1.35	1.45	1.51	1.39	1.37	1.36
Mean Value of 37 Comp.		1.36	1.43	1.56	1.41	1.34	1.35
Standard Dev. of 37 Comp.		0.42	0.46	0.60	0.47	0.39	0.40

Table 3-8
BM3 Model
Modal Time History vs. Method 1
Response Spectrum (14 modes plus missing mass)
Support Reactions

Forces in lbs
Moments in in-lbs
1 % Damping

NODE NO.	REAC. TYPE	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	Fx	43.71	1.10	1.10	1.10	1.10	1.10	1.10
1	Fy	4.36	1.26	1.34	1.70	1.41	1.15	1.18
1	Fz	1.60	4.74	4.87	6.63	5.33	4.10	4.27
1	Mx	49.88	5.55	5.72	7.86	6.28	4.76	4.99
1	My	776.40	0.89	0.89	0.93	0.90	0.88	0.88
1	Mz	278.42	0.78	0.82	0.95	0.84	0.75	0.75
4	Fx	116.79	0.80	0.80	0.81	0.80	0.80	0.80
4	Fz	20.01	3.77	3.89	5.30	4.26	3.26	3.41
7	Fy	13.27	1.20	1.29	1.37	1.27	1.18	1.18
11	Fy	13.31	1.48	1.55	1.84	1.59	1.42	1.44
11	Fz	81.34	0.99	1.00	1.15	1.03	0.95	0.96
15	Fx	731.47	0.60	0.61	0.63	0.61	0.61	0.61
17	Fy	25.60	1.91	1.95	2.35	1.98	1.85	1.88
17	Fz	65.36	1.22	1.30	1.41	1.25	1.23	1.22
36	Fy	46.69	1.93	2.17	2.28	2.03	1.92	1.92
36	Fz	42.12	2.02	2.36	2.51	2.11	2.04	2.03
38	Fx	732.18	0.89	0.89	0.89	0.89	0.89	0.89
38	Fy	43.44	1.21	1.24	1.26	1.23	1.21	1.21
38	Fz	29.95	1.40	1.60	1.71	1.47	1.47	1.45
38	Mx	719.05	1.95	2.25	2.39	2.07	1.93	1.94
38	My	2084.97	1.31	1.50	1.62	1.38	1.37	1.36
38	Mz	3085.87	1.20	1.22	1.25	1.21	1.20	1.20
23	Fx	259.59	1.02	1.05	1.10	1.03	1.03	1.02
23	Fy	26.08	4.04	5.06	5.56	4.43	3.96	3.99
31	Fx	55.05	0.92	0.92	0.93	0.92	0.92	0.92
31	Fy	14.17	1.75	1.92	2.02	1.82	1.77	1.76
31	Fz	16.08	1.97	2.34	2.38	2.08	1.88	1.91
31	Mx	1137.30	2.15	2.64	2.66	2.28	2.02	2.07
31	My	612.38	0.74	0.80	0.83	0.76	0.75	0.75
31	Mz	1773.20	0.79	0.85	0.88	0.82	0.79	0.80
Mean Value of 30 Comp.			1.72	1.86	2.14	1.84	1.64	1.66
Standard Dev. of 30 Comp.			1.23	1.34	1.81	1.41	1.06	1.10

Table 3-9
BM3 Model
Modal Time History vs. Method 1
Response Spectrum (14 modes plus missing mass)
Pipe End Moments

Moments in in-lbs $(M_x^2+M_y^2+M_z^2)^{1/2}$ 1 % Damping

ELEM. NO.	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	790.95	0.91	0.92	0.99	0.93	0.89	0.90
2	836.77	0.87	0.88	0.93	0.89	0.86	0.87
3	1429.28	0.96	0.98	1.15	1.01	0.91	0.92
4	763.87	1.07	1.09	1.22	1.12	1.05	1.06
5	597.98	1.65	1.68	2.04	1.77	1.53	1.56
6	2133.18	0.97	0.98	1.03	0.99	0.96	0.96
7	1877.81	0.98	0.98	1.01	0.99	0.97	0.97
8	852.74	2.70	2.82	3.70	3.03	2.39	2.47
9	1415.21	2.04	2.09	2.61	2.22	1.85	1.90
10	1946.24	1.08	1.09	1.23	1.12	1.04	1.05
11	3829.46	0.90	0.91	0.92	0.91	0.90	0.90
12	8428.48	0.80	0.80	0.80	0.80	0.79	0.79
13	6741.03	0.86	0.86	0.87	0.86	0.85	0.85
14	13219.91	0.59	0.60	0.63	0.61	0.60	0.60
15	2293.75	1.50	1.55	1.72	1.55	1.53	1.53
16	1696.61	1.99	2.09	2.37	2.08	2.06	2.03
17	2455.62	1.84	1.90	2.29	1.92	1.84	1.84
18	2363.76	1.23	1.27	1.36	1.26	1.23	1.23
19	1625.77	1.46	1.50	1.68	1.50	1.46	1.46
20	7544.81	1.26	1.33	1.37	1.28	1.26	1.26
21	1590.16	1.52	1.64	1.70	1.56	1.52	1.52
22	2046.41	1.46	1.56	1.64	1.51	1.49	1.49
23	1807.75	1.61	1.71	1.81	1.66	1.64	1.63
24	1518.31	1.63	1.76	1.85	1.69	1.67	1.65
25	922.51	1.26	1.41	1.47	1.32	1.24	1.25
26	677.35	1.51	1.72	1.80	1.60	1.48	1.50
27	390.71	1.99	2.31	2.43	2.11	1.95	1.96
28	737.42	1.33	1.40	1.44	1.36	1.33	1.33
29	830.76	1.22	1.27	1.29	1.24	1.23	1.22
30	1994.56	1.42	1.70	1.72	1.50	1.37	1.39
31	7428.79	1.65	1.78	1.88	1.71	1.68	1.67
32	8785.32	1.46	1.60	1.69	1.52	1.49	1.48
33	10970.06	1.18	1.32	1.40	1.23	1.21	1.20
34	5993.82	1.22	1.24	1.26	1.24	1.23	1.23
35	6788.22	1.20	1.25	1.29	1.22	1.21	1.20
36	1874.84	1.62	1.77	1.86	1.67	1.64	1.63
37	3501.06	1.37	1.48	1.54	1.41	1.40	1.39
Mean Value of 37 Comp.		1.36	1.44	1.57	1.42	1.34	1.35
Standard Dev. of 37 Comp.		0.42	0.46	0.60	0.47	0.39	0.40

Table 3-10
BM3 Model
Modal Time History vs. Method 2
Response Spectrum (14 modes using Lindley-Yow approach plus missing mass)

Forces in lbs | Support Reactions | 1 % Damping
Moments in in-lbs

NODE NO.	REAC. TYPE	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	Fx	43.71	1.06	1.06	1.06	1.06	1.06	1.06
1	Fy	4.36	0.85	0.87	0.98	0.89	0.81	0.82
1	Fz	1.60	2.21	2.28	3.02	2.47	1.94	2.02
1	Mx	49.88	2.50	2.60	3.52	2.82	2.15	2.25
1	My	776.40	0.89	0.89	0.90	0.89	0.89	0.89
1	Mz	278.42	0.83	0.84	0.87	0.85	0.83	0.83
4	Fx	116.79	0.90	0.90	0.91	0.91	0.90	0.90
4	Fz	20.01	1.81	1.87	2.45	2.02	1.61	1.66
7	Fy	13.27	1.05	1.07	1.10	1.07	1.04	1.05
11	Fy	13.31	1.14	1.17	1.29	1.19	1.13	1.13
11	Fz	81.34	0.87	0.87	0.91	0.88	0.86	0.86
15	Fx	731.47	0.81	0.81	0.81	0.81	0.81	0.81
17	Fy	25.60	1.42	1.44	1.62	1.46	1.39	1.40
17	Fz	65.36	0.97	0.99	1.02	0.98	0.97	0.97
36	Fy	46.69	1.35	1.44	1.47	1.39	1.35	1.35
36	Fz	42.12	1.28	1.42	1.49	1.32	1.30	1.29
38	Fx	732.18	1.05	1.05	1.05	1.05	1.05	1.05
38	Fy	43.44	1.10	1.11	1.12	1.11	1.10	1.10
38	Fz	29.95	1.27	1.31	1.34	1.28	1.28	1.28
38	Mx	719.05	1.25	1.38	1.43	1.30	1.24	1.24
38	My	2084.97	1.27	1.30	1.33	1.28	1.28	1.28
38	Mz	3085.87	1.10	1.10	1.11	1.10	1.10	1.10
23	Fx	259.59	1.32	1.32	1.33	1.32	1.32	1.32
23	Fy	26.08	2.14	2.57	2.80	2.31	2.09	2.11
31	Fx	55.05	1.02	1.03	1.03	1.02	1.02	1.02
31	Fy	14.17	1.26	1.33	1.36	1.29	1.26	1.26
31	Fz	16.08	1.43	1.64	1.65	1.48	1.38	1.40
31	Mx	1137.30	1.49	1.79	1.80	1.57	1.41	1.44
31	My	612.38	0.99	1.00	1.01	0.99	0.99	0.99
31	Mz	1773.20	1.00	1.00	1.01	1.00	1.00	1.00
Mean Value of 30 Comp.			1.25	1.31	1.43	1.30	1.22	1.23
Standard Dev. of 30 Comp.			0.42	0.48	0.67	0.49	0.35	0.37

Table 3-11
BM3 Model
Modal Time History vs. Method 2
Response Spectrum (14 modes using Lindley-Yow approach plus missing mass)
Pipe End Moments

Moments in in-lbs $(M_x^2+M_y^2+M_z^2)^{1/2}$ **1 % Damping**

ELEM. NO.	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	790.95	0.90	0.90	0.91	0.90	0.89	0.89
2	836.77	0.90	0.90	0.91	0.90	0.89	0.89
3	1429.28	0.91	0.91	0.95	0.92	0.90	0.90
4	763.87	0.98	0.98	1.02	0.99	0.97	0.98
5	597.98	1.20	1.21	1.32	1.24	1.17	1.18
6	2133.18	0.94	0.94	0.95	0.94	0.92	0.94
7	1877.81	0.95	0.95	0.96	0.95	0.94	0.94
8	852.74	1.59	1.64	1.96	1.72	1.49	1.52
9	1415.21	1.42	1.44	1.60	1.47	1.37	1.39
10	1946.24	0.97	0.97	1.01	0.98	0.96	0.96
11	3829.46	0.91	0.91	0.91	0.91	0.91	0.91
12	8428.48	0.86	0.86	0.86	0.86	0.86	0.86
13	6741.03	0.89	0.89	0.90	0.89	0.89	0.89
14	13219.91	0.81	0.81	0.81	0.81	0.81	0.81
15	2293.75	1.20	1.22	1.28	1.22	1.21	1.21
16	1696.61	1.42	1.46	1.56	1.46	1.44	1.44
17	2455.62	1.38	1.40	1.57	1.41	1.38	1.38
18	2363.76	1.09	1.10	1.13	1.10	1.09	1.09
19	1625.77	1.22	1.23	1.31	1.24	1.22	1.22
20	7544.81	1.11	1.12	1.14	1.11	1.11	1.11
21	1590.16	1.30	1.33	1.36	1.31	1.30	1.30
22	2046.41	1.49	1.50	1.52	1.49	1.49	1.49
23	1807.75	1.38	1.41	1.44	1.39	1.38	1.38
24	1518.31	1.42	1.45	1.48	1.44	1.43	1.43
25	922.51	1.11	1.18	1.20	1.13	1.10	1.11
26	677.35	1.20	1.29	1.31	1.23	1.18	1.19
27	390.71	1.56	1.68	1.73	1.60	1.53	1.55
28	737.42	1.24	1.26	1.27	1.25	1.24	1.24
29	830.76	1.18	1.20	1.20	1.19	1.18	1.18
30	1994.56	1.26	1.39	1.39	1.29	1.23	1.25
31	7428.79	1.58	1.61	1.63	1.60	1.58	1.58
32	8785.32	1.46	1.49	1.52	1.47	1.46	1.46
33	10970.06	1.26	1.30	1.32	1.27	1.27	1.27
34	5993.82	1.13	1.14	1.14	1.14	1.14	1.14
35	6788.22	1.23	1.24	1.25	1.23	1.23	1.23
36	1874.84	1.31	1.35	1.38	1.33	1.32	1.32
37	3501.06	1.25	1.28	1.29	1.27	1.25	1.25
Mean Value of 37 Comp.		1.19	1.21	1.26	1.21	1.18	1.19
Standard Dev. of 37 Comp.		0.22	0.24	0.28	0.24	0.22	0.22

Table 3-12
BM3 Model
Modal Time History vs. Method 2
Response Spectrum (31 modes using Lindley-Yow approach plus missing mass)

Forces in lbs | Support Reactions | 1 % Damping
Moments in in-lbs

NODE NO.	REAC. TYPE	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	Fx	43.71	1.06	1.06	1.06	1.06	1.06	1.06
1	Fy	4.36	0.91	0.94	1.05	0.96	0.88	0.89
1	Fz	1.60	2.22	2.29	3.03	2.48	1.95	2.03
1	Mx	49.88	2.51	2.61	3.52	2.83	2.16	2.26
1	My	776.40	0.89	0.89	0.90	0.89	0.89	0.89
1	Mz	278.42	0.86	0.87	0.89	0.87	0.85	0.85
4	Fx	116.79	0.91	0.91	0.91	0.91	0.91	0.91
4	Fz	20.01	1.82	1.87	2.46	2.02	1.61	1.67
7	Fy	13.27	1.05	1.07	1.10	1.07	1.04	1.05
11	Fy	13.31	1.14	1.18	1.29	1.19	1.13	1.13
11	Fz	81.34	0.87	0.87	0.91	0.88	0.86	0.86
15	Fx	731.47	0.81	0.81	0.81	0.81	0.81	0.81
17	Fy	25.60	1.42	1.44	1.62	1.46	1.39	1.40
17	Fz	65.36	0.97	0.99	1.02	0.98	0.97	0.97
36	Fy	46.69	1.35	1.44	1.47	1.39	1.35	1.35
36	Fz	42.12	1.29	1.42	1.50	1.33	1.31	1.30
38	Fx	732.18	1.05	1.05	1.05	1.05	1.05	1.05
38	Fy	43.44	1.10	1.11	1.12	1.11	1.10	1.10
38	Fz	29.95	1.27	1.31	1.34	1.28	1.28	1.28
38	Mx	719.05	1.25	1.38	1.43	1.30	1.24	1.24
38	My	2084.97	1.27	1.30	1.33	1.28	1.28	1.28
38	Mz	3085.87	1.10	1.10	1.11	1.10	1.10	1.10
23	Fx	259.59	1.32	1.32	1.33	1.32	1.32	1.32
23	Fy	26.08	2.14	2.57	2.80	2.31	2.09	2.11
31	Fx	55.05	1.03	1.03	1.03	1.03	1.03	1.03
31	Fy	14.17	1.26	1.33	1.36	1.29	1.26	1.26
31	Fz	16.08	1.43	1.64	1.65	1.48	1.38	1.40
31	Mx	1137.30	1.49	1.79	1.80	1.57	1.41	1.44
31	My	612.38	0.99	1.01	1.01	1.00	0.99	0.99
31	Mz	1773.20	1.00	1.01	1.02	1.00	1.00	1.00
Mean Value of 30 Comp.			1.26	1.32	1.43	1.31	1.22	1.23
Standard Dev. of 30 Comp.			0.42	0.48	0.67	0.49	0.35	0.37

Table 3-13
BM3 Model
Modal Time History vs. Method 2
Response Spectrum (31 modes using Lindley-Yow approach plus missing mass)
Pipe End Moments

Moments in in-lbs $(M_x^2+M_y^2+M_z^2)^{1/2}$ **1 % Damping**

ELEM. NO.	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	790.95	0.90	0.90	0.91	0.90	0.89	0.90
2	836.77	0.90	0.90	0.91	0.90	0.89	0.89
3	1429.28	0.91	0.92	0.96	0.93	0.90	0.90
4	763.87	0.98	0.98	1.02	0.99	0.97	0.98
5	597.98	1.20	1.21	1.32	1.24	1.17	1.18
6	2133.18	0.94	0.94	0.95	0.94	0.94	0.94
7	1877.81	0.95	0.95	0.96	0.95	0.94	0.94
8	852.74	1.59	1.64	1.96	1.72	1.49	1.52
9	1415.21	1.42	1.44	1.60	1.47	1.37	1.39
10	1946.24	0.97	0.97	1.01	0.98	0.96	0.96
11	3829.46	0.91	0.91	0.91	0.91	0.91	0.91
12	8428.48	0.86	0.86	0.86	0.86	0.86	0.86
13	6741.03	0.89	0.89	0.90	0.89	0.89	0.89
14	13219.91	0.81	0.81	0.81	0.81	0.81	0.81
15	2293.75	1.20	1.22	1.28	1.22	1.21	1.21
16	1696.61	1.42	1.46	1.56	1.46	1.44	1.44
17	2455.62	1.38	1.40	1.57	1.41	1.38	1.38
18	2363.76	1.09	1.10	1.13	1.10	1.09	1.09
19	1625.77	1.22	1.23	1.31	1.24	1.22	1.22
20	7544.81	1.11	1.12	1.14	1.11	1.11	1.11
21	1590.16	1.30	1.33	1.36	1.31	1.30	1.30
22	2046.41	1.49	1.50	1.52	1.50	1.49	1.49
23	1807.75	1.38	1.41	1.44	1.39	1.38	1.38
24	1518.31	1.42	1.45	1.48	1.44	1.43	1.43
25	922.51	1.12	1.19	1.21	1.15	1.11	1.12
26	677.35	1.21	1.30	1.32	1.24	1.19	1.20
27	390.71	1.56	1.70	1.73	1.60	1.53	1.55
28	737.42	1.24	1.26	1.27	1.25	1.24	1.24
29	830.76	1.19	1.20	1.21	1.19	1.19	1.19
30	1994.56	1.27	1.39	1.40	1.30	1.23	1.25
31	7428.79	1.58	1.61	1.63	1.60	1.58	1.58
32	8785.32	1.46	1.49	1.52	1.47	1.46	1.46
33	10970.06	1.26	1.30	1.32	1.27	1.27	1.27
34	5993.82	1.13	1.14	1.14	1.14	1.14	1.14
35	6788.22	1.23	1.24	1.25	1.23	1.23	1.23
36	1874.84	1.31	1.35	1.39	1.33	1.32	1.32
37	3501.06	1.25	1.28	1.29	1.27	1.25	1.25
Mean Value of 37 Comp.		1.19	1.22	1.26	1.21	1.18	1.19
Standard Dev. of 37 Comp.		0.22	0.24	0.28	0.24	0.22	0.22

Table 3-14
BM3 Model
Modal Time History vs. Method 3
Response Spectrum (14 modes using modified spectrum approach plus static ZPA)

Forces in lbs — Support Reactions — 1 % Damping
Moments in in-lbs

NODE NO.	REAC. TYPE	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	Fx	43.71	1.06	1.06	1.06	1.06	1.06	1.06
1	Fy	4.36	0.85	0.87	0.98	0.89	0.81	0.82
1	Fz	1.60	2.21	2.28	3.01	2.47	1.94	2.02
1	Mx	49.88	2.50	2.60	3.52	2.82	2.16	2.25
1	My	776.40	0.89	0.89	0.90	0.89	0.89	0.89
1	Mz	278.42	0.83	0.84	0.87	0.85	0.83	0.83
4	Fx	116.79	0.90	0.90	0.91	0.91	0.90	0.90
4	Fz	20.01	1.81	1.87	2.45	2.01	1.61	1.66
7	Fy	13.27	1.05	1.07	1.10	1.07	1.04	1.05
11	Fy	13.31	1.14	1.17	1.29	1.18	1.12	1.13
11	Fz	81.34	0.87	0.87	0.91	0.88	0.86	0.86
15	Fx	731.47	0.81	0.81	0.81	0.81	0.81	0.81
17	Fy	25.60	1.42	1.43	1.61	1.45	1.39	1.40
17	Fz	65.36	0.97	0.99	1.02	0.98	0.97	0.97
36	Fy	46.69	1.35	1.43	1.47	1.38	1.34	1.34
36	Fz	42.12	1.28	1.41	1.49	1.32	1.30	1.29
38	Fx	732.18	1.05	1.05	1.05	1.05	1.05	1.05
38	Fy	43.44	1.10	1.11	1.12	1.11	1.10	1.10
38	Fz	29.95	1.27	1.31	1.34	1.28	1.28	1.28
38	Mx	719.05	1.24	1.37	1.42	1.29	1.23	1.23
38	My	2084.97	1.27	1.30	1.33	1.28	1.28	1.28
38	Mz	3085.87	1.10	1.11	1.11	1.10	1.10	1.10
23	Fx	259.59	1.32	1.32	1.33	1.32	1.32	1.32
23	Fy	26.08	2.12	2.56	2.78	2.29	2.07	2.09
31	Fx	55.05	1.02	1.03	1.03	1.02	1.02	1.02
31	Fy	14.17	1.26	1.33	1.36	1.28	1.26	1.26
31	Fz	16.08	1.42	1.63	1.64	1.48	1.37	1.39
31	Mx	1137.30	1.50	1.79	1.80	1.57	1.41	1.44
31	My	612.38	0.99	1.00	1.01	0.99	0.99	0.99
31	Mz	1773.20	1.00	1.00	1.01	1.00	1.00	1.00
Mean Value of 30 Comp.			1.25	1.31	1.42	1.30	1.22	1.23
Standard Dev. of 30 Comp.			0.42	0.48	0.67	0.49	0.35	0.37

Table 3-15
BM3 Model
Modal Time History vs. Method 3
Response Spectrum (14 modes using modified spectrum approach plus static ZPA)
Pipe End Moments

Moments in in-lbs $(M_x^2+M_y^2+M_z^2)^{1/2}$ 1 % Damping

ELEM. NO.	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	790.95	0.90	0.90	0.91	0.90	0.89	0.89
2	836.77	0.90	0.90	0.91	0.90	0.89	0.89
3	1429.28	0.91	0.91	0.95	0.92	0.90	0.90
4	763.87	0.98	0.98	1.02	0.99	0.97	0.98
5	597.98	1.20	1.21	1.31	1.24	1.17	1.17
6	2133.18	0.94	0.94	0.95	0.94	0.92	0.92
7	1877.81	0.95	0.95	0.96	0.95	0.94	0.95
8	852.74	1.59	1.64	1.96	1.72	1.49	1.52
9	1415.21	1.42	1.45	1.61	1.49	1.37	1.39
10	1946.24	0.97	0.97	1.01	0.98	0.96	0.96
11	3829.46	0.91	0.91	0.92	0.91	0.91	0.91
12	8428.48	0.86	0.86	0.86	0.86	0.86	0.86
13	6741.03	0.89	0.89	0.90	0.89	0.89	0.89
14	13219.91	0.81	0.81	0.81	0.81	0.81	0.81
15	2293.75	1.20	1.22	1.28	1.22	1.21	1.21
16	1696.61	1.42	1.46	1.56	1.46	1.44	1.44
17	2455.62	1.38	1.40	1.57	1.41	1.38	1.38
18	2363.76	1.09	1.10	1.13	1.10	1.09	1.09
19	1625.77	1.22	1.23	1.30	1.24	1.22	1.22
20	7544.81	1.11	1.12	1.14	1.11	1.11	1.11
21	1590.16	1.30	1.33	1.34	1.31	1.29	1.30
22	2046.41	1.48	1.50	1.52	1.49	1.49	1.49
23	1807.75	1.38	1.41	1.43	1.39	1.38	1.38
24	1518.31	1.42	1.45	1.48	1.43	1.42	1.42
25	922.51	1.11	1.18	1.20	1.13	1.10	1.10
26	677.35	1.20	1.29	1.31	1.23	1.18	1.19
27	390.71	1.55	1.68	1.73	1.60	1.52	1.53
28	737.42	1.24	1.26	1.27	1.25	1.24	1.24
29	830.76	1.18	1.19	1.20	1.19	1.18	1.18
30	1994.56	1.26	1.39	1.40	1.29	1.23	1.25
31	7428.79	1.58	1.61	1.63	1.60	1.58	1.58
32	8785.32	1.46	1.49	1.51	1.47	1.46	1.46
33	10970.06	1.26	1.30	1.32	1.27	1.27	1.27
34	5993.82	1.13	1.14	1.14	1.14	1.14	1.14
35	6788.22	1.23	1.24	1.25	1.23	1.23	1.23
36	1874.84	1.31	1.35	1.39	1.33	1.32	1.32
37	3501.06	1.25	1.28	1.30	1.27	1.25	1.25
Mean Value of 37 Comp.		1.19	1.22	1.26	1.21	1.18	1.19
Standard Dev. of 37 Comp.		0.22	0.24	0.28	0.24	0.22	0.22

Table 3-16
BM3 Model
Modal Time History vs. Method 3
Response Spectrum (14 modes using modified spectrum approach plus static ZPA)

Forces in lbs | Support Reactions | 5 % Damping
Moments in in-lbs

NODE NO.	REAC. TYPE	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	Fx	44.22	1.05	1.05	1.05	1.05	1.05	1.05
1	Fy	3.86	0.76	0.79	0.94	0.94	0.65	0.65
1	Fz	0.97	3.35	3.43	4.61	4.56	1.70	1.74
1	Mx	18.89	6.03	6.20	8.57	8.41	2.41	2.49
1	My	694.80	0.97	0.97	0.98	0.98	0.96	0.96
1	Mz	240.22	0.86	0.87	0.90	0.93	0.86	0.86
4	Fx	113.70	0.92	0.92	0.92	0.92	0.92	0.92
4	Fz	13.85	2.43	2.47	3.29	3.25	1.34	1.36
7	Fy	7.95	1.21	1.25	1.29	1.39	1.13	1.14
11	Fy	7.13	1.55	1.61	1.80	1.94	1.40	1.41
11	Fz	71.89	0.95	0.95	0.99	1.00	0.92	0.92
15	Fx	690.97	0.84	0.84	0.84	0.85	0.85	0.85
17	Fy	20.14	1.55	1.57	1.73	1.78	1.42	1.43
17	Fz	60.05	0.98	1.00	1.04	1.05	0.99	0.99
36	Fy	38.70	1.38	1.46	1.50	1.58	1.29	1.30
36	Fz	42.33	1.19	1.27	1.32	1.35	1.19	1.20
38	Fx	740.68	1.04	1.04	1.04	1.04	1.04	1.04
38	Fy	38.18	1.20	1.20	1.21	1.22	1.19	1.19
38	Fz	29.85	1.23	1.26	1.30	1.31	1.30	1.30
38	Mx	465.42	1.49	1.63	1.72	1.85	1.35	1.35
38	My	2080.83	1.24	1.27	1.29	1.30	1.29	1.29
38	Mz	2713.20	1.19	1.20	1.20	1.22	1.19	1.19
23	Fx	270.50	1.25	1.26	1.26	1.27	1.26	1.26
23	Fy	10.58	3.83	4.76	5.33	5.88	2.69	2.76
31	Fx	54.46	1.04	1.04	1.04	1.04	1.04	1.04
31	Fy	9.76	1.72	1.77	1.81	1.88	1.73	1.74
31	Fz	8.66	2.12	2.35	2.38	2.49	1.89	1.90
31	Mx	560.28	2.21	2.59	2.61	2.69	1.80	1.82
31	My	582.28	1.01	1.02	1.02	1.03	1.01	1.01
31	Mz	1685.20	1.03	1.04	1.04	1.05	1.03	1.03
Mean Value of 30 Comp.			1.59	1.67	1.87	1.91	1.30	1.31
Standard Dev. of 30 Comp.			1.10	1.21	1.65	1.67	0.45	0.47

Table 3-17
BM3 Model
Modal Time History vs. Method 3
Response Spectrum (14 modes using modified spectrum approach plus static ZPA)
Pipe End Moments

Moments in in-lbs $(M_x^2+M_y^2+M_z^2)^{1/2}$ 5 % Damping

ELEM. NO.	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) SRSS	GROUPING	TEN PER.	NRC-DSC	RB-DSC	CQC
1	713.05	0.96	0.96	0.97	0.98	0.95	0.95
2	765.02	0.96	0.96	0.97	0.97	0.95	0.95
3	1323.65	0.95	0.95	0.99	0.99	0.92	0.92
4	684.42	1.02	1.02	1.05	1.06	1.00	1.00
5	528.66	1.25	1.26	1.38	1.39	1.14	1.14
6	1893.29	1.00	1.00	1.01	1.02	0.99	0.99
7	1664.64	1.01	1.01	1.02	1.03	1.01	1.01
8	764.93	1.58	1.62	1.96	1.98	1.18	1.19
9	1319.86	1.35	1.36	1.53	1.54	1.18	1.18
10	1714.12	1.00	1.00	1.04	1.05	0.97	0.97
11	3507.84	0.96	0.96	0.97	0.97	0.96	0.96
12	7650.43	0.92	0.92	0.92	0.93	0.92	0.92
13	6075.21	0.95	0.95	0.96	0.96	0.95	0.95
14	12487.74	0.83	0.83	0.84	0.85	0.84	0.84
15	2030.00	1.26	1.28	1.33	1.35	1.30	1.30
16	1057.35	2.08	2.12	2.25	2.31	2.17	2.18
17	1495.69	2.01	2.03	2.23	2.26	1.96	1.97
18	2027.05	1.20	1.21	1.23	1.26	1.19	1.19
19	1370.50	1.34	1.35	1.40	1.43	1.32	1.32
20	6612.74	1.21	1.23	1.24	1.25	1.21	1.21
21	1337.38	1.44	1.47	1.48	1.52	1.42	1.43
22	1803.08	1.63	1.64	1.66	1.68	1.64	1.64
23	1352.91	1.75	1.78	1.81	1.84	1.78	1.78
24	1229.48	1.67	1.70	1.73	1.75	1.70	1.70
25	958.32	1.02	1.05	1.06	1.08	1.00	1.00
26	689.97	1.09	1.14	1.16	1.19	1.04	1.05
27	322.88	1.67	1.77	1.81	1.89	1.60	1.60
28	624.60	1.39	1.40	1.41	1.44	1.38	1.38
29	737.73	1.29	1.29	1.30	1.32	1.29	1.29
30	1784.26	1.24	1.32	1.33	1.34	1.17	1.18
31	7194.98	1.56	1.58	1.60	1.62	1.58	1.58
32	8768.03	1.40	1.42	1.44	1.46	1.42	1.42
33	10912.71	1.22	1.24	1.26	1.27	1.24	1.24
34	5338.54	1.22	1.22	1.23	1.24	1.22	1.22
35	6348.71	1.27	1.28	1.28	1.29	1.27	1.27
36	1755.84	1.32	1.35	1.37	1.40	1.34	1.34
37	3263.83	1.29	1.31	1.32	1.34	1.30	1.30
Mean Value of 37 Comp.		1.28	1.30	1.34	1.36	1.26	1.26
Standard Dev. of 37 Comp.		0.30	0.32	0.35	0.36	0.31	0.31

Table 3-18
Equivalence of R_r
Lindley-Yow Plus Missing Mass vs. Static ZPA

Node No.	Reac. Type	Eqn. 2-12 n=8	Missing Mass n > 8	Eqn. 2-12 n=14	Missing Mass n > 14	Eqn. 2-12 n=31	Missing Mass n > 31	Total R_r n=8	Total R_r n=14	Total R_r n=31	Static ZPA (Note 1)
1	Fx	1.40	-47.60	1.95	-48.20	6.47	-52.70	-46.20	-46.25	-46.23	-46.21
1	Fy	0.42	0.24	1.35	-0.71	-11.90	12.50	0.66	0.64	0.60	0.66
1	Fz	0.05	0.22	1.30	-1.04	0.27	0.00	0.27	0.26	0.27	0.26
1	Mx	23.30	-12.80	-18.10	28.50	10.60	-0.27	10.50	10.40	10.33	10.53
1	My	-290.00	-38.90	-347.00	18.50	-329.00	0.30	-328.90	-328.50	-328.70	-329.25
1	Mz	64.40	94.20	102.00	56.60	67.50	90.60	158.60	158.60	158.10	158.43
4	Fx	-10.10	-92.70	-13.60	-89.20	-53.90	-48.90	-102.80	-102.80	-102.80	-102.75
4	Fz	-1.19	-4.29	-12.90	7.43	-4.95	-0.48	-5.48	-5.47	-5.43	-5.49
7	Fy	-1.07	1.05	-3.70	3.69	-0.31	0.30	-0.02	-0.01	-0.01	-0.01
11	Fy	-2.68	1.50	-0.80	-0.40	-1.20	0.00	-1.18	-1.20	-1.20	-1.17
11	Fz	30.40	7.26	49.40	-11.80	37.40	0.20	37.66	37.60	37.60	37.65
15	Fx	-236.00	-238.00	-448.00	-25.60	-456.00	-18.40	-474.00	-473.60	-474.40	-474.26
17	Fy	-2.17	5.97	3.72	0.08	3.81	-0.01	3.80	3.80	3.80	3.78
17	Fz	16.80	-52.00	-32.10	-3.05	-33.90	-1.15	-35.20	-35.15	-35.05	-35.17
36	Fy	45.00	-36.10	9.33	-0.41	7.65	1.27	8.90	8.92	8.92	8.92
36	Fz	26.60	5.19	65.30	-33.60	42.20	-10.60	31.79	31.70	31.60	31.68
38	Fx	9.05	-776.00	-137.00	-630.00	-146.00	-621.00	-766.95	-767.00	-767.00	-767.29
38	Fy	-16.90	5.67	-11.70	0.55	-11.30	0.08	-11.23	-11.15	-11.22	-11.24
38	Fz	1.16	-32.30	-48.10	16.90	-33.20	2.02	-31.14	-31.20	-31.18	-31.17
38	Mx	400.00	-590.00	-183.00	-5.94	-190.00	0.99	-190.00	-188.94	-189.01	-189.06
38	My	114.00	-2320.00	-3130.00	928.00	-2280.00	79.30	-2206.00	-2202.00	-2200.70	-2202.83
38	Mz	1190.00	-392.00	837.00	-33.00	806.00	-3.17	798.00	804.00	802.83	804.58
23	Fx	34.40	-338.00	-153.00	-152.00	-304.00	-0.54	-303.60	-305.00	-304.54	-304.11
23	Fy	55.40	-56.00	1.71	-2.39	-0.74	0.05	-0.60	-0.68	-0.69	-0.68
31	Fx	4.04	-60.50	-7.24	-49.30	-23.30	-33.20	-56.46	-56.54	-56.50	-56.51
31	Fy	-5.69	5.42	-1.19	0.97	-0.08	-0.15	-0.27	-0.22	-0.23	-0.25
31	Fz	12.30	-10.10	-0.05	2.29	2.21	0.03	2.20	2.24	2.24	2.24
31	Mx	1040.00	-818.00	78.00	138.00	215.00	1.31	222.00	216.00	216.31	219.27
31	My	-74.30	655.00	231.00	350.00	577.00	4.16	580.70	581.00	581.16	580.84
31	Mz	-289.00	1990.00	549.00	1150.00	1700.00	5.43	1701.00	1699.00	1705.43	1702.48

Note 1: Dynamic model mass distribution used in static ZPA calculation

BM3 Piping Model

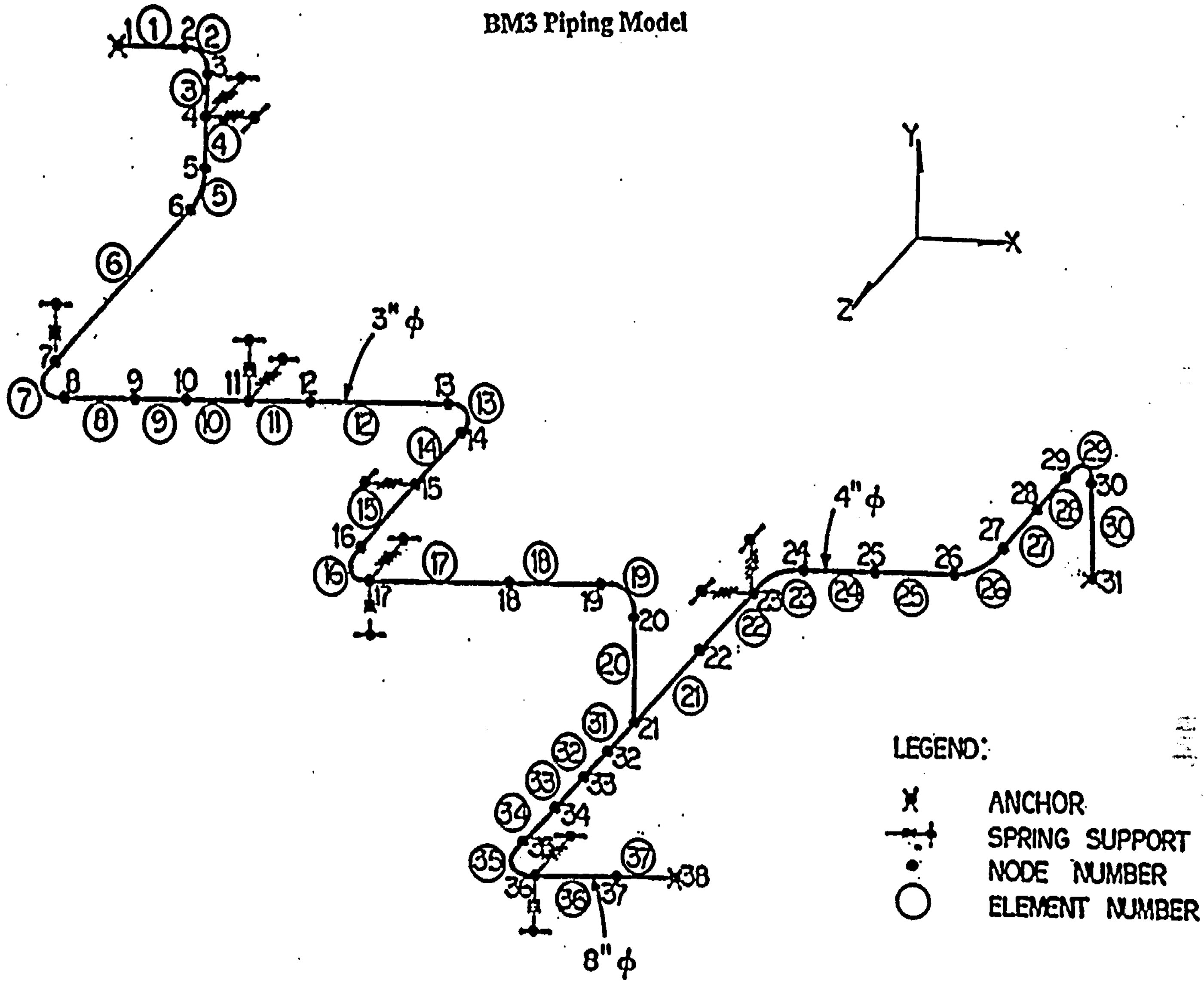


Figure 3-1

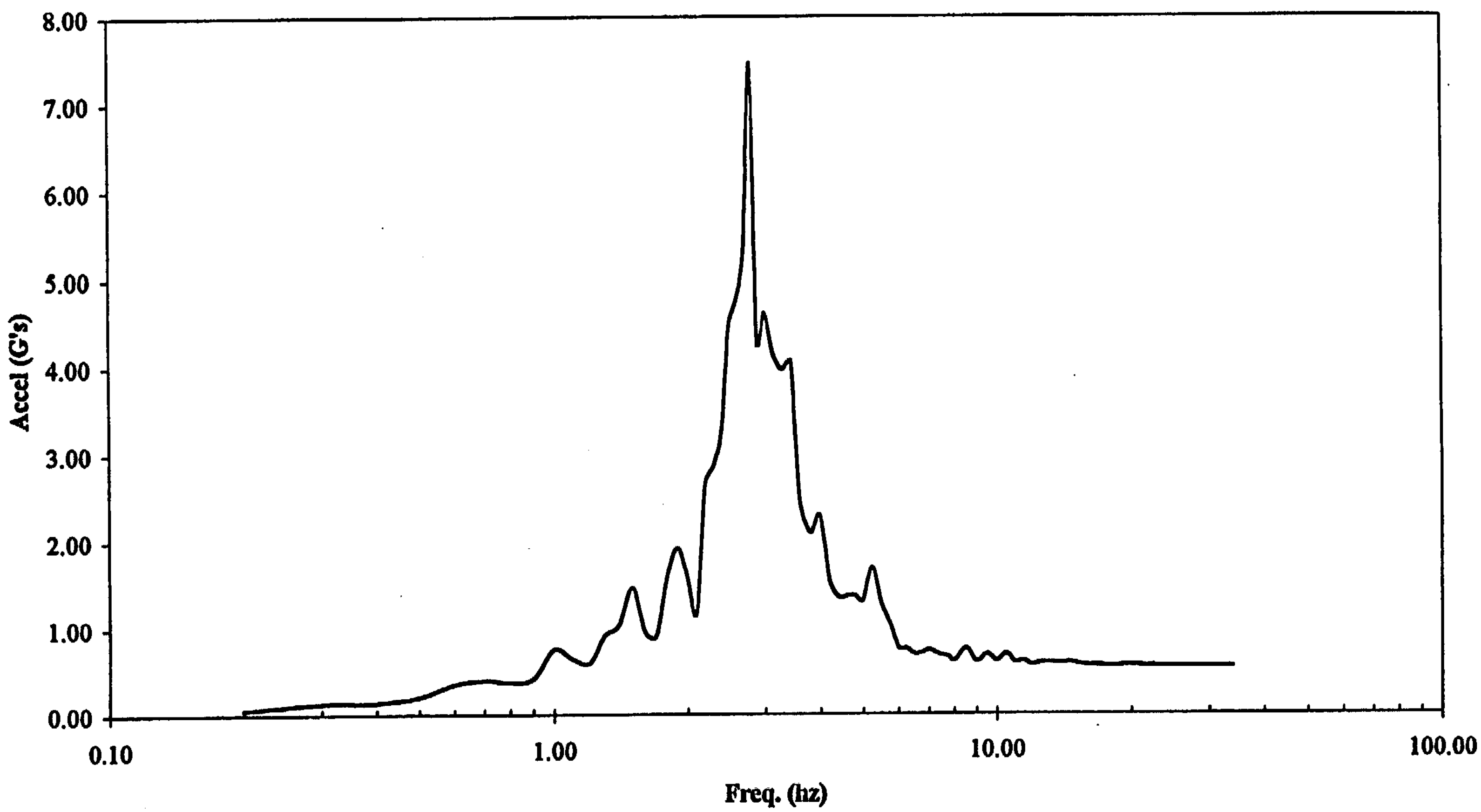


Figure 3-2 BM3 Input Spectrum (1%)

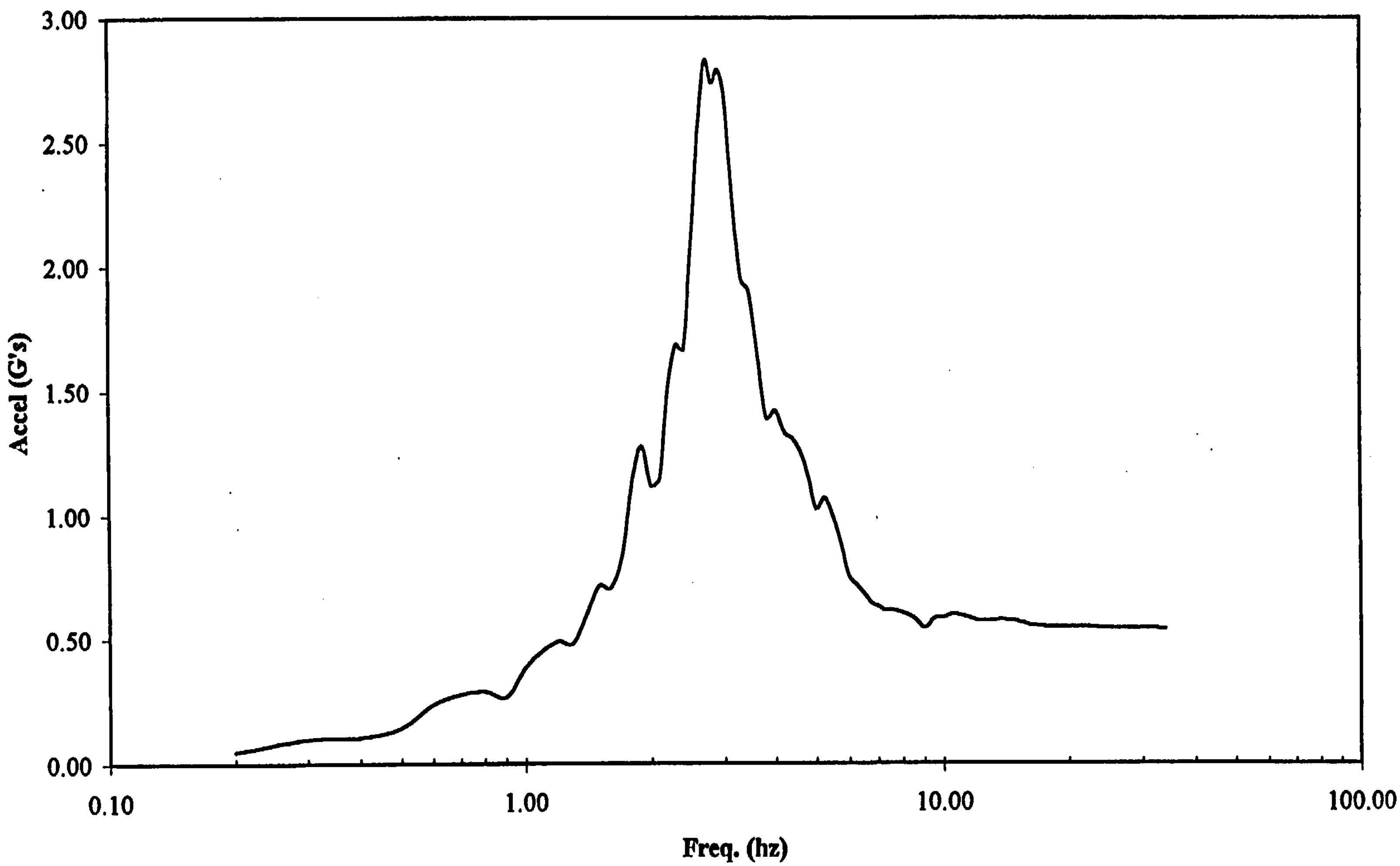


Figure 3-3 BM3 Input Spectrum (5%)

4 SUPPLEMENTARY STUDY OF ADDITIONAL MODAL RESPONSE COMBINATION METHODS

4.1 Basis for Study of Additional Modal Response Combination Methods

Although the agreement between the Response Spectrum Analysis (RSA) results and the Time History Solution is generally acceptable from an engineering perspective, the M_x support reaction at Node 1 provides an excellent basis for a detailed investigation of the effects of RSA approximations on the prediction of peak response. It is noted that the magnitude of the reaction is relatively small when compared to the maximum moment reaction ($\approx$2%), and the RSA results exceed the Time History Solution by a factor of 2 or greater. Therefore, the inaccuracy of RSA in this case is not significant from a design standpoint. However, the specific elements involved in generating this significant overprediction were deemed worthy of a more detailed investigation.

It is important to recall that f_{ZPA} = 16.5 Hz was initially chosen based on the response spectrum, and that the first fourteen (14) natural modes of vibration for BM3 have frequencies below 16.5 Hz. Modal contributions above 16.5 Hz are accounted for by the missing mass contribution. Therefore, the evaluation initially concentrated on the combination of the first 14 modes and comparison to the 14 mode time history solution.

A numerical study was performed to examine the individual modal contributions of the first fourteen (14) modes in both the time history solution and the RSA solution. For Node 1, M_x an interesting peculiarity of the modal contributions to the total predicted response is that two modes, Mode 9 (11.06Hz) and Mode 11 (11.50Hz), dominate the response. The responses are opposite in sign, and close in magnitude (-154 vs. 219), with identical times of peak response in the Time History Solution. This type of modal behavior poses a difficult challenge to the RSA methodologies employed in the initial study phase.

Examination of Tables 3-2 and 3-4 for Node 1, M_x shows the deterioration in accuracy between the 8 mode combination and the 14 mode combination. The SRSS combination of modal responses produces a gross overprediction. The DSC and CQC modal combination approaches should improve the situation. However, for 1% damping, $f_1 \approx$ 10Hz, and frequency ratio $\approx$ 0.95, the mode correlation coefficient is 0.15-0.23 (see Appendix D). This provides only a minor correction. The largest overpredictions are generated by the NRC Grouping, Ten Percent and DSC Methods, in which closely spaced modes are combined by absolute sum. The complete solution utilizing the Lindley-Yow assumption (see Tables 3-10, 3-12, or 3-14) provides improvement because significant fractions of the modal responses for modes 9 and 11 are included in the algebraic (in-phase) summation, and therefore tend to cancel each other. However, the fractions of the mode 9 and 11 responses included in the out-of-phase response combination are still significant enough to introduce inaccuracy on the conservative side.

4.2 Description of Additional Modal Response Combination Methods

The behavior of the RSA solution for the Node 1, M_x reaction led to a recommendation by the Project Peer Reviewers that a sensitivity study be conducted using alternate modal response combination methods, which might produce more accurate results. Two alternate methods were selected based on the specific behavior at Node 1, M_x. The Gupta Method was also selected for inclusion in this sensitivity study. In the Gupta Method, f_2 defines the frequency above which the modal responses are assumed to be perfectly correlated and consequently combined algebraically. Three different assumptions for frequency f_2 were tested.

The mathematical formulations for the additional modal response combination methods are presented below. The two new methods, developed in conjunction with the project Peer Reviewers, are designated GROUP-1 and GROUP-2. Applications of Gupta's Method are designated GUP-0, GUP-1 and GUP-2. For comparison, the Lindley-Yow CQC results are also included in the tabulated results under the heading LINDLEY. Terms used in the following sections were previously defined in Chapter 2.

4.2.1 Grouping Methods 1 and 2

Grouping Methods consist of two similar procedures, designated GROUP-1 and GROUP-2. GROUP-1 was proposed by Dr. R.P. Kennedy following his peer review of the initial numerical results. GROUP-2 is a variation of GROUP-1 in which the mode 8 through mode 14 responses are not split into out-of-phase and in-phase response components.

Group-1

The 14 modes ($f_i < f_{ZPA}$) are first divided into their in-phase and out-of-phase response components according to the Lindley-Yow method. This has been previously described in Section 2.2.1.

The out-of-phase modal response components are combined in accordance with the following formula:

$$Rp = \sqrt{Rp_i^2 + Rp_2^2 + (Rp_3 + Rp_4)^2 + (Rp_5 + Rp_6)^2 + Rp_7^2 + (Rp_8 + Rp_9 + Rp_{10} + Rp_{11} + Rp_{12} + Rp_{13} + Rp_{14})^2}$$

The in-phase modal response components are algebraically summed:

$$Rr = \sum_{i=1}^{14} Rr_i$$

The total response for the first 14 modes is:

$$Rt = \sqrt{Rp^2 + Rr^2}$$

which assumes that the in-phase and out-of-phase modal response components *Rr* and *Rp*, are uncorrelated.

Group-2

The first 7 modes ($f_i < 9.8$Hz) are divided by the Lindley-Yow method. The out-of-phase response components are combined in accordance with the following formula:

$$Rp = \sqrt{Rp_1^2 + Rp_2^2 + (Rp_3 + Rp_4)^2 + (Rp_5 + Rp_6)^2 + Rp_7^2}$$

which is identical to GROUP-1 for the first 7 modes. The in-phase response components for the first 7 modes are algebraically summed with the responses for modes 8 through 14, which are treated as completely in-phase with the input time history:

$$Rr = \sum_{i=1}^{7} Rr_i + \sum_{i=8}^{14} R_i$$

The total response for the first 14 modes is again given by:

$$R_t = \sqrt{Rp^2 + Rr^2}$$

GROUP-1 and GROUP-2 differ only in their treatment of modes 8 through 14. GROUP-1 and GROUP-2 both assume that all modes above 9.8 Hz (3.5 times the frequency of the spectral peak) are in-phase and combine algebraically. However, GROUP-1 maintains the Lindley-Yow split for all modes with $f < f_{ZPA}$.

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A detailed evaluation of the 1% damping, response spectrum generation calculation concluded that SDOF oscillators with frequencies above 9.5 Hz respond in-phase with the input time history. This is documented in Appendix F. The Appendix F procedure can be applied to any response spectrum, in order to pinpoint the frequency (designated f_{IP}) above which SDOF oscillators respond in-phase with the input time history. Knowledge of f_{IP} for a given response spectrum provides a definitive separation between out-of-phase modal responses for which a combination rule is needed and the in-phase modal responses for which algebraic combination is appropriate.

A significant characteristic of GROUP-1 and GROUP-2 is that 100% correlation is assumed between closely spaced modes. The modal correlation coefficients are assumed to be either zero or one, similar to the NRC Grouping Method. However, algebraic combination is employed instead of the absolute sum combination used in the NRC Grouping Method. Grouping of modes in GROUP-1 and GROUP-2 is problem dependent; it cannot be viewed as a generally applicable alternative for the treatment of closely spaced modes.

The primary basis for the grouping is the shape of the BM3 input response spectrum. The single well defined, narrowly banded peak in the response spectrum at 2.8 Hz is the result of strong filtering of the ground motion through the building structure/soil. This spectral shape is encountered frequently in the analysis of distributed systems (e.g., piping) attached to building structure. The strong filtering produces a dynamic input to the distributed system which more closely resembles a single-frequency excitation rather than a broad spectral input characteristic of seismic ground motion. The CQC and DSC mode combination formulations for closely spaced modes are rooted in random vibration theory, which is more appropriate to earthquake ground motion. The phasing of closely spaced modal responses for strongly filtered input motion may be more analogous to the response of MDOF systems to a single-frequency input motion. The basis for the grouping of modes in GROUP-1 and GROUP-2 can be defined as follows:

- The peak of the response spectrum occurs at 2.8 Hz
- Modes with frequencies less than 1.75 times the peak (1.75 x 2.8 Hz = 4.9 Hz) have uncorrelated out-of-phase response components
- Modes with frequencies greater than 3.5 times the peak (3.5 x 2.8 Hz = 9.8 Hz) respond in-phase
- Between 1.75 and 3.5 times the peak frequency, out-of-phase modal response components are grouped and combined algebraically if they are closely spaced (i.e., within 10% of the lowest frequency mode comprising a group). This implies they are perfectly correlated.

The selection of 1.75 and 3.5 times the peak frequency for division of the frequency band is subject to further study and refinement.

4.2.2 Gupta Method - Variations to Value of f_2

The Gupta Method was previously described in Section 2.2.3. The total in-phase response and total out-of-phase response are obtained by:

$$Rr = \sum_{i=1}^{14} Rr_i$$

$$Rp = \left[\sum_{j=1}^{14}\sum_{k=1}^{14} C_{jk} Rp_j Rp_k\right]^{1/2}$$

C_{jk} is defined by Eq. 3200-19 of ASCE Draft Standard 4 (Reference 10):

$$C_{jk} = \frac{2\sqrt{\beta_j \beta_k}}{(\beta_j + \beta_k)}\left[1 + \left(\frac{f_j - f_k}{\beta_j f_j + \beta_k f_k}\right)^2\right]^{-1}$$

where f_j, f_k are the natural frequency and β_j, β_k are damping ratio for the jth and kth mode respectively. This equation is essentially equivalent to the CQC method. See Section 2.1.7.

The total response for the first fourteen (14) modes is obtained by SRSS combination of Rp and Rr:

$$Rt = \sqrt{Rp^2 + Rr^2}$$

The Gupta Method is well documented (Reference 4) and has been recommended by the ASCE (Reference 10). In the initial phase of this study, it was decided not to include the Gupta Method because it uses semi-empirical relationships developed from earthquake ground motion records, which may not be applicable to building-filtered seismic motion. However, the follow-up sensitivity study provided an opportunity to include several variations of the Gupta Method. The key parameter varied was f_2, which defines the frequency above which modal responses are combined algebraically (i.e., $\alpha = 1.0$). GUP-0 assumes f_2 equals 3 times the frequency at the spectral peak (3 x 2.8 Hz = 8.4 Hz). GUP-1 uses Gupta's definition of f_2 from Reference 4 (see Section 2.2.3). This yields a numerical value $f_2 = 11.9$ Hz. GUP-2 uses the definition of $f_2 = f_{ZPA}$; therefore, $f_2 = 16.5$ Hz. The selection of f_2 has a significant impact on the predicted response.

4.3 Discussion of Numerical Results for Additional Modal Response Combination Methods

The results of the mode response combination sensitivity study are tabulated in Table 4-1 and 4-2, without the missing mass contribution above 14 modes, and Table 4-3 and 4-4, with the missing mass contribution above 14 modes. The missing mass contribution is algebraically summed with the in-phase component of response from the first 14 modes before combination by SRSS with the out-of-phase component of response. The last two rows of the tables provide the mean value of the response ratio and the corresponding standard deviation.

All of the alternate combination methods except GUP-2 produced more accurate results than Lindley-Yow/CQC. GUP-2 produced the most conservatives results on average, and had the largest standard deviation. This is noteworthy because GUP-2 and Lindley-Yow/CQC use essentially equivalent definitions for C_{jk}, and setting $f_2 = f_{ZPA}$ in the Gupta Method is equivalent to the Lindley-Yow definition of f_{IP}. The only real difference is the numerical value of α as a function of frequency, in the range f_{SP} to f_{ZPA} .

GROUP-1, GROUP-2, and GUP-0 produced results in agreement with each other at most locations. The mean of the ratios and standard deviation of the ratios are the best of the methods tested. It is important to note that the onset of completely in-phase response is assumed to occur between modes 7 and 8, in all three methods (see Table 3-1). GUP-1, with $f_2 = 11.9$ Hz, assumes completely in-phase response occurs between modes 11 and 12 (see Table 3-1). GUP-1 corresponds to Gupta's Method as presented in Reference 4 and described in Section 2.2.3. For the BM3 model and seismic input, GUP-1 produces superior correlation to the time history solution, compared to Lindley-Yow/CQC.

The trend toward improvement in correlation to the time history solution with decreasing $f_{IP}(f_2)$ indicates that appropriate estimation of $f_{IP} < f_{ZPA}$ is a significant aspect of RSA. One method to identify f_{IP} is provided in Appendix F.

The largest discrepancies among the different methods occur for support reactions which are "orthogonal" to the x-directional load input: F_y, F_z, M_x. These reactions are typically smaller in magnitude than the primary reactions F_x, M_y, M_z . For the primary reactions, there is very little scatter in the results from one method to another. The 14 mode solution and the complete solution exhibit similar correlation with the corresponding time history solutions. For the "orthogonal" reactions, considerable scatter between methods is exhibited for both the 14 mode and complete solutions.

Examining the results for Node 1, M_x, which initially instigated the follow-up sensitivity study, it is evident that the RSA prediction is extremely sensitive to the specific modal response combination method employed. Of particular surprise was the deterioration in accuracy exhibited by GROUP-1, GROUP-2, and GUP-0 between the 14 mode solution and the complete solution. Further investigation of the time history solution for Node 1, M_x revealed that the time of maximum response shifts as successive modal contributions are added. At the end of the 14 mode summation, the maximum response occurs at 13.88 seconds. From Table E-1, it is noted that the complete solution for Node 1, M_x maximizes at 6.68 seconds.

The missing mass contribution above 14 modes occurs at the time of and is scaled to the peak acceleration of the input time history: -.54g at 7.35 seconds. The reaction at Node 1, M_x due to missing mass above 14 modes is of the same order as the 14 mode time history summation: 28.54 vs. 43.23. However, they occur at 13.88 seconds and 7.35 seconds respectively, while the complete time history solution maximizes at 6.68 seconds. It is obvious that the phase relationship between these two components of the time history solution is very important in arriving at the peak response of the complete solution. This element is lacking in the RSA method regardless of the specific mode combination method employed.

As indicated by the scatter of predictions by RSA in the initial study phase and in the follow-up sensitivity study for Node 1, M_x, there will be cases where RSA cannot produce a reasonably accurate prediction of the peak response. However, it is important to note that this appears to be a problem for the "orthogonal" response quantities (i.e., F_y, F_z, M_x for an x-direction input). A typical design RSA would include three directions of input motion, with the major F_y, F_z and M_x contributions coming from y and z direction input. In the SRSS combination of directional responses, errors in F_y, F_z and M_x due to x-direction input should have only a minor effect on the total predicted response due to three directional loading.

An investigation of the modal response combination methods for three directions of input motion was not considered as part of this study, because the introduction of additional variables would have complicated the evaluation of the combination methods. This would be a logical and very useful extension of the results reported herein.

Appendix H presents independent results generated by Professors A.K. Gupta and Abhinav Gupta of North Carolina State University for three directional loading of the BM3 model. These results verify the supposition stated above. Large errors in "orthogonal" output quantities (e.g., Node 1, M_x) are not significant in the combined response to three directional loading.

Appendix H also documents a proposed alternate approach to determine f_2 (f_{IP}) for use in the Gupta Method. The proposed method is qualitative and requires some judgment, but has an advantage over the quantitative method proposed in Appendix F. For the Appendix F approach, response spectra generation analyses are needed to determine f_{IP} as a function of damping ratio. The method proposed in Appendix H requires only the response spectra plots for different damping values. A single value for f_2, independent of damping, is estimated from the superimposed plots, and is defined as follows: "The key frequency f_2 is the minimum frequency at which spectra at various damping values converge." It appears that the Appendix H method would require unbroadened spectra for the determination of f_2.

For the BM3 x-direction seismic input, $f_2 = 6.0$ Hz is estimated in Appendix H. The method of Appendix F, identified

$$f_{IP} = 9.5 \text{ Hz @ 1\% damping}$$
$$f_{IP} = 7.5 \text{ Hz @ 5\% damping}$$

for the BM3 seismic input.

Since accurate definition of f_{IP} is essential to improving the correlation between RSA solutions and time history solutions, the methods proposed in Appendix F and Appendix H warrant additional study and refinement. The Appendix F approach is judged to provide an upper bound to f_{IP}; the Appendix H method, based on its application to the current problem, may provide a lower bound to f_{IP}. It is noted that for 1% damping, GUP-0 ($f_2 = 8.4$ Hz) provided a better match to the time history, on average, than the solution presented in Appendix H ($f_2 = 6.0$ Hz). See Table 4-3 and Appendix H Table 3.

4.4 Summary of Results

Based on the supplementary quantitative study described above, the following observations are made:

- For highly-filtered, in-structure response spectra, improvement in accuracy can be attained by better definition of the onset of completely in-phase modal response (f_{IP}).
- Algebraic grouping of closely-spaced modes may be more applicable to highly-filtered response spectra than the RB-DSC and CQC methods.
- Large inaccuracies in prediction of small magnitude responses, due to one direction of seismic input, will likely be insignificant in the total RSA solution for combined three direction of seismic input.
- Candidate methodologies for definition of f_{IP} have been proposed; these warrant further investigation and refinement.
- The response of MDOF systems to a single frequency input may provide a mathematical basis for combining out-of-phase modal response components for highly-filtered, in-structure response spectra. Additional study in this area is warranted.

- The flexibility inherent in Gupta's method, to specify any value of f_{IP} (through definition of f_2) is an advantage over the Lindley-Yow method. The sensitivity of the results to change in f_2 highlights the importance of improving the definition of f_{IP}.

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Table 4-1
BM3 Model
14 Mode Combination
Time History vs. Response Spectrum
Support Reactions Comparison Among Different Methods

Forces in lbs **1 % Damping**
Moments in in-lbs

NODE NO.	REAC. TYPE	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) GROUP-1	GROUP-2	GUP-0	GUP-1	GUP-2	LINDLEY
1	Fx	3.96	0.84	0.85	0.85	0.82	0.81	0.85
1	Fy	3.51	0.96	0.96	0.94	0.97	1.15	1.08
1	Fz	2.57	0.75	0.76	0.77	0.96	1.71	1.35
1	Mx	43.23	0.81	0.72	0.74	1.49	3.51	2.63
1	My	794.03	0.87	0.88	0.88	0.87	0.87	0.88
1	Mz	215.80	0.87	0.88	0.87	0.86	0.88	0.90
4	Fx	29.24	0.83	0.83	0.84	0.82	0.82	0.84
4	Fz	27.64	0.74	0.75	0.75	0.92	1.60	1.28
7	Fy	13.87	1.04	1.04	1.00	1.04	1.06	1.04
11	Fy	13.25	1.15	1.15	1.01	1.18	1.27	1.13
11	Fz	93.01	0.80	0.81	0.82	0.80	0.80	0.83
15	Fx	706.21	0.81	0.82	0.84	0.78	0.73	0.80
17	Fy	25.56	1.42	1.42	1.27	1.53	1.65	1.41
17	Fz	62.29	0.99	0.96	0.90	1.01	1.11	0.99
36	Fy	46.89	1.15	1.13	1.07	1.26	1.51	1.35
36	Fz	68.17	1.13	1.13	1.16	1.11	1.13	1.16
38	Fx	121.91	1.33	1.32	1.32	1.34	1.26	1.25
38	Fy	43.98	1.08	1.08	1.07	1.10	1.14	1.09
38	Fz	42.65	1.32	1.31	1.32	1.32	1.23	1.24
38	Mx	713.03	1.00	0.86	0.77	1.07	1.43	1.26
38	My	2783.00	1.32	1.32	1.32	1.33	1.24	1.25
38	Mz	3117.80	1.08	1.08	1.07	1.10	1.13	1.09
23	Fx	159.21	1.46	1.43	1.43	1.50	1.47	1.39
23	Fy	28.00	1.01	0.97	0.69	1.43	2.44	1.97
31	Fx	6.99	1.30	1.26	1.26	1.41	1.46	1.33
31	Fy	13.95	1.24	1.16	1.18	1.27	1.42	1.29
31	Fz	15.50	1.17	1.17	1.26	1.52	1.68	1.45
31	Mx	1102.30	1.10	1.10	1.24	1.60	1.80	1.49
31	My	259.45	1.13	1.09	1.06	1.18	1.18	1.09
31	Mz	608.00	1.15	1.11	1.10	1.27	1.31	1.17
Mean Value of 30 Comp.			1.06	1.04	1.03	1.16	1.36	1.23
Standard Dev. of 30 Comp.			0.20	0.20	0.21	0.25	0.54	0.36

Table 4-2
BM3 Model
14 Mode Combination
Time History vs. Response Spectrum
Pipe End Moments Comparison Among Different Methods

Moments in in-lbs $(M_x^2+M_y^2+M_z^2)^{1/2}$ **1 % Damping**

ELEM. NO.	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) GROUP-1	GROUP-2	GUP-0	GUP-1	GUP-2	LINDLEY
1	776.55	0.88	0.89	0.89	0.88	0.89	0.90
2	759.36	0.89	0.89	0.89	0.88	0.88	0.89
3	968.28	0.87	0.87	0.88	0.88	0.98	0.94
4	773.63	0.94	0.95	0.95	0.94	0.97	0.97
5	701.53	0.97	0.97	0.97	0.98	1.10	1.04
6	2207.60	0.90	0.90	0.91	0.90	0.90	0.91
7	1948.20	0.91	0.92	0.92	0.92	0.92	0.92
8	837.44	1.22	1.20	1.17	1.28	1.79	1.56
9	1357.40	1.28	1.27	1.27	1.31	1.57	1.44
10	2183.80	0.89	0.90	0.90	0.89	0.90	0.91
11	4173.00	0.87	0.88	0.88	0.86	0.84	0.87
12	8283.10	0.86	0.87	0.87	0.85	0.84	0.86
13	6579.90	0.89	0.90	0.90	0.89	0.88	0.89
14	13174.00	0.81	0.82	0.84	0.78	0.72	0.80
15	2294.00	1.24	1.15	1.14	1.22	1.33	1.22
16	1693.10	1.50	1.35	1.29	1.49	1.69	1.45
17	2456.50	1.39	1.37	1.26	1.49	1.61	1.38
18	2377.20	1.08	1.08	1.06	1.10	1.15	1.09
19	1635.50	1.22	1.21	1.18	1.26	1.33	1.22
20	7586.00	1.10	1.09	1.08	1.12	1.17	1.11
21	1616.20	1.30	1.29	1.28	1.34	1.40	1.32
22	1977.10	1.48	1.45	1.44	1.52	1.54	1.45
23	1900.20	1.49	1.46	1.45	1.53	1.54	1.45
24	1607.70	1.54	1.52	1.51	1.58	1.58	1.50
25	265.36	0.90	0.88	1.20	1.64	2.16	1.77
26	271.57	1.11	1.08	1.28	1.69	2.14	1.77
27	348.19	1.29	1.25	1.28	1.52	1.73	1.52
28	552.58	1.30	1.24	1.23	1.34	1.43	1.33
29	581.00	1.26	1.19	1.18	1.28	1.36	1.27
30	1102.30	1.30	1.29	1.41	1.77	1.96	1.64
31	7586.80	1.63	1.59	1.58	1.66	1.66	1.59
32	9093.40	1.49	1.47	1.45	1.51	1.50	1.45
33	11281.00	1.30	1.29	1.28	1.32	1.29	1.27
34	6061.80	1.15	1.14	1.14	1.17	1.19	1.15
35	6634.10	1.25	1.24	1.24	1.27	1.26	1.23
36	2451.60	1.24	1.23	1.23	1.26	1.25	1.23
37	4009.70	1.26	1.25	1.25	1.27	1.26	1.23
Mean Value of 37 Comp.		1.16	1.14	1.15	1.23	1.32	1.23
Standard Dev. of 37 Comp.		0.23	0.22	0.21	0.29	0.38	0.28

Table 4-3
BM3 Model
14 Modes Plus Missing Mass Combination
Time History vs. Response Spectrum
Support Reactions Comparison Among Different Methods

Forces in lbs — 1 % Damping
Moments in in-lbs

NODE NO.	REAC. TYPE	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES) GROUP-1	GROUP-2	GUP-0	GUP-1	GUP-2	LINDLEY
1	Fx	43.71	1.06	1.06	1.06	1.06	1.07	1.06
1	Fy	4.36	0.72	0.72	0.71	0.74	0.90	0.82
1	Fz	1.60	0.91	0.83	0.84	1.22	2.63	2.03
1	Mx	49.88	0.64	0.38	0.39	1.10	3.00	2.26
1	My	776.40	0.88	0.89	0.89	0.88	0.88	0.89
1	Mz	278.42	0.81	0.81	0.81	0.79	0.79	0.83
4	Fx	116.79	0.90	0.90	0.90	0.89	0.87	0.90
4	Fz	20.01	0.83	0.80	0.81	1.08	2.14	1.67
7	Fy	13.27	1.05	1.05	1.01	1.06	1.09	1.05
11	Fy	13.31	1.14	1.14	1.02	1.18	1.27	1.13
11	Fz	81.34	0.83	0.84	0.84	0.83	0.86	0.86
15	Fx	731.47	0.81	0.82	0.84	0.78	0.73	0.80
17	Fy	25.60	1.42	1.42	1.27	1.52	1.65	1.41
17	Fz	65.36	0.97	0.94	0.88	1.00	1.08	0.97
36	Fy	46.69	1.15	1.14	1.08	1.26	1.52	1.35
36	Fz	42.12	1.23	1.22	1.20	1.21	1.39	1.30
38	Fx	732.18	1.05	1.07	1.07	1.07	1.04	1.05
38	Fy	43.44	1.09	1.09	1.08	1.11	1.15	1.10
38	Fz	29.95	1.42	1.33	1.34	1.36	1.29	1.28
38	Mx	719.05	1.00	0.85	0.76	1.06	1.42	1.25
38	My	2084.97	1.41	1.34	1.34	1.36	1.30	1.28
38	Mz	3085.87	1.09	1.08	1.08	1.11	1.15	1.10
23	Fx	259.59	1.35	1.38	1.38	1.42	1.36	1.33
23	Fy	26.08	1.08	1.02	0.72	1.54	2.62	2.11
31	Fx	55.05	1.03	1.04	1.04	1.05	1.04	1.03
31	Fy	14.17	1.21	1.14	1.16	1.26	1.40	1.27
31	Fz	16.08	1.14	1.15	1.24	1.48	1.62	1.40
31	Mx	1137.30	1.08	1.08	1.23	1.56	1.74	1.45
31	My	612.38	0.99	1.02	1.00	1.04	1.00	0.98
31	Mz	1773.20	0.99	1.01	1.00	1.04	1.02	0.99
Mean Value of 30 Comp.			1.04	1.02	1.00	1.14	1.37	1.23
Standard Dev. of 30 Comp.			0.20	0.22	0.22	0.23	0.56	0.37

Table 4-4
BM3 Model
14 Modes Plus Missing Mass Combination
Time History vs. Response Spectrum
Pipe End Moments Comparison Among Different Methods

Moments in in-lbs $(M_x^2+M_y^2+M_z^2)^{1/2}$ 1 % Damping

ELEM. NO.	TIME HISTORY (TH) VALUE	RESPONSE SPECTRUM (EXPRESSED AS RATIOS OF THE TH VALUES)					
		GROUP-1	GROUP-2	GUP-0	GUP-1	GUP-2	LINDLEY
1	790.95	0.88	0.88	0.89	0.87	0.88	0.89
2	836.77	0.88	0.89	0.89	0.87	0.87	0.89
3	1429.28	0.86	0.86	0.86	0.85	0.88	0.89
4	763.87	0.95	0.95	0.95	0.95	0.98	0.97
5	597.98	1.09	1.09	1.09	1.13	1.28	1.18
6	2133.18	0.92	0.92	0.92	0.92	0.93	0.93
7	1877.81	0.93	0.94	0.94	0.94	0.95	0.94
8	852.74	1.20	1.18	1.15	1.25	1.76	1.53
9	1415.21	1.24	1.23	1.23	1.27	1.51	1.39
10	1946.24	0.93	0.94	0.94	0.93	0.96	0.96
11	3829.46	0.90	0.91	0.91	0.90	0.89	0.91
12	8428.48	0.86	0.86	0.87	0.85	0.83	0.86
13	6741.03	0.88	0.89	0.89	0.88	0.87	0.88
14	13219.91	0.81	0.82	0.84	0.78	0.72	0.80
15	2293.75	1.24	1.15	1.14	1.22	1.33	1.22
16	1696.61	1.50	1.35	1.29	1.49	1.69	1.45
17	2455.62	1.39	1.37	1.26	1.49	1.61	1.38
18	2363.76	1.09	1.08	1.07	1.11	1.15	1.09
19	1625.77	1.22	1.21	1.18	1.26	1.33	1.23
20	7544.81	1.11	1.10	1.09	1.13	1.17	1.12
21	1590.16	1.28	1.27	1.25	1.33	1.39	1.30
22	2046.41	1.52	1.51	1.50	1.58	1.57	1.49
23	1807.75	1.43	1.37	1.36	1.45	1.49	1.39
24	1518.31	1.47	1.42	1.41	1.49	1.52	1.43
25	922.51	1.03	1.02	1.06	1.08	1.14	1.12
26	677.35	1.06	1.03	1.08	1.13	1.26	1.19
27	390.71	1.37	1.33	1.37	1.55	1.71	1.55
28	737.42	1.22	1.21	1.20	1.28	1.31	1.24
29	830.76	1.18	1.17	1.16	1.22	1.24	1.19
30	1994.56	1.12	1.14	1.17	1.32	1.38	1.25
31	7428.79	1.63	1.59	1.58	1.66	1.67	1.59
32	8785.32	1.51	1.48	1.47	1.53	1.52	1.47
33	10970.06	1.31	1.30	1.29	1.33	1.30	1.28
34	5993.82	1.14	1.13	1.12	1.16	1.18	1.14
35	6788.22	1.25	1.25	1.24	1.27	1.26	1.23
36	1874.84	1.34	1.28	1.27	1.33	1.38	1.32
37	3501.06	1.29	1.26	1.25	1.29	1.30	1.26
Mean Value of 37 Comp.		1.16	1.15	1.14	1.19	1.25	1.19
Standard Dev. of 37 Comp.		0.23	0.21	0.20	0.24	0.29	0.22

5 CONCLUSIONS AND RECOMMENDATIONS

5.1 Technical Conclusions

Based on the results of this evaluation the following technical conclusions have been developed:

1) Rosenblueth's DSC method and Der Kiureghian's CQC method for combining the out-of-phase modal response components (Rp_i) represent a technical improvement over SRSS and the current methods accepted in Regulatory Guide 1.92 for treatment of closely spaced modes.

2) Separation of the out-of-phase (Rp_i) and the in-phase (Rr_i) modal response components for amplified modes ($f < f_{ZPA}$), in accordance with the Lindley-Yow method and the Gupta method (see Method 2, Section 2.4.2), provides a more accurate RSA prediction than the traditionally applied method, which assumes out-of-phase response ($Rp_i = R_i$) for amplified modes (see Method 1, Section 2.4.1).

3) The contribution from high frequency modes ($f \geq f_{ZPA}$) is accurately calculated by the Missing Mass method and should be included in all RSA. The Static ZPA method is a convenient way to account for the high frequency modes when implementing the Lindley-Yow method (see Method 3, Section 2.4.3)..

4) Specific structural systems with significant participation of low frequency modes ($f < f_{\text{spectral peak}}$) were not analyzed. The Lindley-Yow method is not suitable for analysis of low frequency systems because of limitations imposed by the definition of the in-phase response component. The Gupta method is not restricted because low frequency modes are correctly treated as out-of-phase (i.e., $Rp_i = R_i$).

5) Numerical evaluation of the highly filtered, in-structure response spectrum applied to the BM3 model indicates that the onset of totally in-phase response ($Rr_i = R_i$) occurs at a frequency (f_{IP}) significantly below f_{ZPA}. The value of f_{IP} can be determined for any response spectrum by the method delineated in Appendix F. This method is potentially applicable to defining f_2 for the Gupta Method (i.e., $f_2 = f_{IP}$). (Note: In Appendix H, Gupta also proposes a new approach to define f_2).

6) Excellent correlation between mode superposition and direct integration time history analysis is achievable for constant modal damping. The missing mass contribution to the mode superposition solution must be included, regardless of the number of modes retained in the solution. Appendix E provides guidelines for each time history analysis method, to improve correlation.

7) The discrete, nodal mass distribution in a dynamic analysis model should be sufficiently refined to produce a static solution in close agreement with a static solution based on continuously distributed mass. Appendix C provides a discussion and guidelines for ensuring an adequate dynamic mass distribution.

8) Broad-banded, ground motion spectral input is more consistent with the technical bases for Rosenblueth's DSC and Der Kiureghian's CQC Methods for modal combination of out-of-phase response components. However, the quantitative results in Chapter 3 indicate their superiority to currently accepted methods, when applied to a highly-filtered, in-structure response spectrum.

9) For highly filtered, in-structure response spectra, improvements to the RSA combination methods evaluated in Chapter 3 appear achievable, based on the supplementary study results presented in Chapter 4. A mathematically based grouping method for highly filtered in-structure response spectrum input and refinement of Gupta's Method, based on new approaches to define f_2 for such spectral input, warrant further evaluation.

10) The BM3 model can potentially serve as a benchmark problem for evaluating alternate methodologies for RSA. Sufficient model, loading, and time history response data is provided in the report.

5.2 Recommendations for Revision to Regulatory Guide 1.92 and Standard Review Plan 3.7.2

Based on the technical conclusions delineated in Section 5.1, the following revisions to regulatory guidance are recommended:

1) The regulatory guidance on modal response combination for RSA should be consolidated in one document. The most logical candidate is R.G. 1.92. This would then be referenced by SRP 3.7.2.

2) Acceptable methodologies for developing the complete RSA solution should be defined in R.G. 1.92.

3) For out-of-phase response combination, Rosenblueth's DSC and Der Kiureghian's CQC methods should be identified in R.G.1.92 as acceptable methods. SRSS is acceptable in the absence of closely spaced modes. The 10% definition for closely spaced modes is applicable only at low damping ratios (≤2%). For higher damping ratios, closely spaced modes should be defined by 5 times the damping ratio (e.g., 25% for 5% damping; 50% for 10% damping). See Section 2.1.4.

The NRC Ten Percent and NRC-DSC methods should be deleted from R.G. 1.92. Because of its extensive prior use, the NRC Grouping method is a candidate for retention in R.G. 1.92, even though absolute summation of closely spaced modes has no technical basis. The maximum damping ratio should be limited to 2%, because the 10% definition of closely spaced modes is not applicable at higher damping levels.

4) For separation of out-of-phase and in-phase components for the amplified modes, the methods of Lindley-Yow and Gupta should be identified in R.G. 1.92 as acceptable methods. R.G. 1.92 should include a cautionary note that the Lindley-Yow method is not suitable for analysis of systems with significant low-frequency response ($f < f_{\text{spectral peak}}$).

5) For the complete RSA solution, Method 2 (Lindley-Yow or Gupta formulation) and Method 3 should be identified in R.G. 1.92 as acceptable methods, subject to the limitations of the Lindley-Yow method cited in (4) above. Method 1 should be strongly discouraged or identified as unacceptable.

6) The missing mass formulation documented in Appendix I should be included in R.G. 1.92, in order to provide a complete definition for Method 2. The contribution of high frequency modes ($f \geq f_{ZPA}$) must be included in all RSA, without exception. Appendix A to SRP 3.7.2 should be deleted.

7) R.G. 1.92 should address the importance of the missing mass contribution in mode superposition time history analysis and define an acceptable procedure, analogous to the approach used in Method 2 for RSA. Only modes with $f < f_{ZPA}$ participate in the modal solution; the missing mass contribution, scaled to the instantaneous input acceleration, is treated as an additional mode in the algebraic summation of modal responses at each time step.

8) SRP 3.7.2 should contain a discussion of dynamic model vs. static model mass distributions, and provide guidelines for ensuring an adequate discrete mass distribution in the dynamic model. Appendix C provides a framework for revision of SRP 3.7.2.

5.3 Recommendations for Additional Evaluation of Modal Response Combination Methods

During the course of this study, a number of areas worthy of additional evaluation were identified. The following recommendations are not restricted to regulatory guidance issues, but also address perceived opportunities for improvement of RSA methodology.

1) Numerical study of the RSA methods should be expanded to include additional problems and three directions of seismic input, in order to quantify the level of correlation

achievable in design-type calculations.

2) The methodologies developed in Appendix F and Appendix H for definition of f_{IP} should be tested for a wide range of spectral shapes and, if proven, should be incorporated as a integral element of the RSA process.

3) A modal response combination method rigorously based on MDOF response to a single frequency input should be developed and tested for highly-filtered, in-structure response spectra.

4) A detailed study of modal response combination methods for systems with significant low-frequency response ($f <$ f spectral peak) should be conducted.

5) The present project did not address independent support motion (ISM) of multiply-supported systems. The applicability of the results and conclusions of this project to improving ISM analysis methods should be investigated.

6 REFERENCES

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14. Lindley, D.W., and Yow, T.R., "Modal Response Summation for Seismic Qualification," Proceedings of the Second ASCE Conference on Civil Engineering and Nuclear Power, Vol. VI, Paper 8-2, Knoxville, TN, September 1980.

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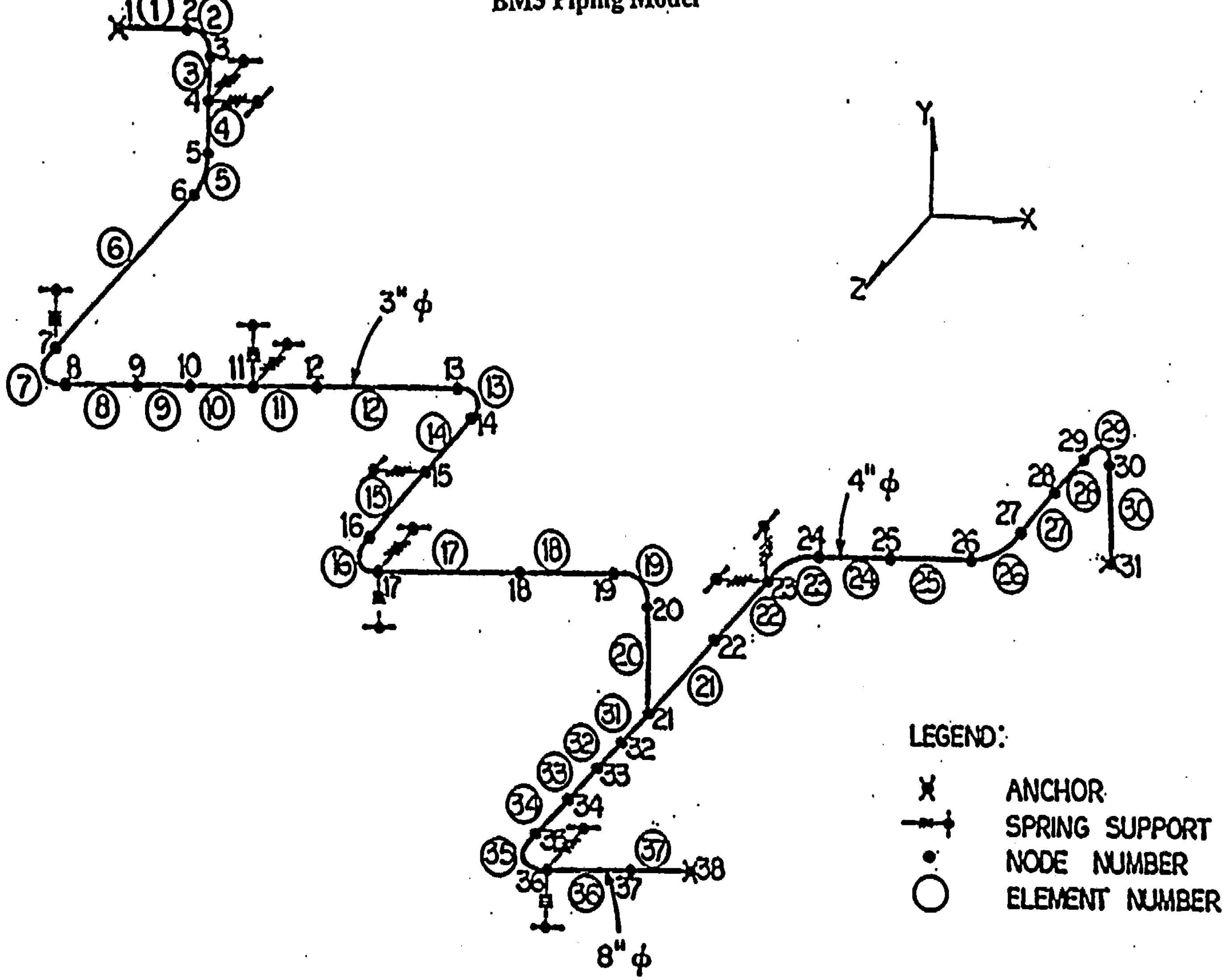
APPENDIX A

DESCRIPTION OF BM3 PIPING MODEL

BM3 Piping Model



449
450
1 MODEL 3---UNIFORM 1 % TIME HISTORY ANALYSIS--- tha_x_mall_d1.inp

C O N T R O L   I N F O R M A T I O N

```
NUMBER OF NODAL POINTS             =   38
NUMBER OF ELEMENT TYPES            =    2
NUMBER OF STATIC LOAD CASES        =    0
NUMBER OF DYNAMIC LOAD CASES       =    1
NUMBER OF ANCHOR MVMT CASES        =    0
NUMBER OF FREQUENCIES              =   31
SOLUTION MODE (MODEX)              =    0
  EQ.0,  EXECUTION
  EQ.1,  DATA CHECK
STRESS CALCULATION FLAG            =    0
   EQ.0 NO
   EQ.1 YES
ASME CODE EVALUATION FLAG          =    0
   EQ.1  CLASS 1 PIPING
   EQ.2  CLASS2 OR CLASS 3 PIPING
ACCELERATION DUE TO GRAVITY        =386.4
BANDWIDTH MINIMIZATION FLAG        =    1
   EQ.0  NO
   EQ.1  YES
ARBITRARY NODE NUMBERING FLAG      =    0
   EQ.0  NO
   EQ.1  YES
NUMBER OF SUPPORT GROUPS           =    1
FLAG FOR NODAL COORD. INPUT UNITS=      0
   EQ.0  CONSISTENT UNIT
   EQ.1  FEET TO INCHES
```

```
 LIST OF ANALYSIS TO BE PERFORMED
 LOAD CASE      DISK FILE      ANALYSIS TYPE

     1              0          UNIFORM TIME HISTORY ANALYSIS
```

1 NODAL POINT INPUT DATA

0NODE	BOUNDARY CONDITION CODES						NODAL POINT COORDINATES			
NUMBER	X	Y	Z	XX	YY	ZZ	X	Y	Z	T
1	0	0	0	0	0	0	.000	.000	.000	.000
2	0	0	0	0	0	0	15.000	.000	.000	.000
3	0	0	0	0	0	0	19.500	-4.500	.000	.000
4	0	0	0	0	0	0	19.500	-180.000	.000	.000
5	0	0	0	0	0	0	19.500	-199.500	.000	.000
6	0	0	0	0	0	0	19.500	-204.000	4.500	.000
7	0	0	0	0	0	0	19.500	-204.000	139.500	.000
8	0	0	0	0	0	0	24.000	-204.000	144.000	.000
9	0	0	0	0	0	0	96.000	-204.000	144.000	.000
10	0	0	0	0	0	0	254.000	-204.000	144.000	.000
11	0	0	0	0	0	0	333.000	-204.000	144.000	.000
12	0	0	0	0	0	0	411.000	-204.000	144.000	.000
13	0	0	0	0	0	0	483.000	-204.000	144.000	.000
14	0	0	0	0	0	0	487.500	-204.000	148.500	.000
15	0	0	0	0	0	0	487.500	-204.000	192.000	.000
16	0	0	0	0	0	0	487.500	-204.000	235.500	.000

17	0	0	0	0	0	0	492.000	-204.000	240.000	.000
18	0	0	0	0	0	0	575.000	-204.000	240.000	.000
19	0	0	0	0	0	0	723.000	-204.000	240.000	.000
20	0	0	0	0	0	0	727.500	-208.500	240.000	.000
21	0	0	0	0	0	0	727.500	-264.000	240.000	.000
22	0	0	0	0	0	0	727.500	-264.000	205.000	.000
23	0	0	0	0	0	0	727.500	-264.000	190.000	.000
24	0	0	0	0	0	0	733.500	-264.000	184.000	.000
25	0	0	0	0	0	0	753.500	-264.000	184.000	.000
26	0	0	0	0	0	0	845.500	-264.000	184.000	.000
27	0	0	0	0	0	0	851.500	-264.000	178.000	.000
28	0	0	0	0	0	0	851.500	-264.000	160.000	.000
29	0	0	0	0	0	0	851.500	-264.000	142.000	.000
30	0	0	0	0	0	0	851.500	-270.000	136.000	.000
31	0	0	0	0	0	0	851.500	-360.000	136.000	.000
32	0	0	0	0	0	0	727.500	-264.000	255.000	.000
33	0	0	0	0	0	0	727.500	-264.000	270.000	.000
34	0	0	0	0	0	0	727.500	-264.000	306.000	.000
35	0	0	0	0	0	0	727.500	-264.000	414.000	.000
36	0	0	0	0	0	0	739.500	-264.000	426.000	.000
37	0	0	0	0	0	0	847.500	-264.000	426.000	.000
38	0	0	0	0	0	0	955.500	-264.000	426.000	.000

1EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	1	2	3	4	5	6
2	7	8	9	10	11	12
3	13	14	15	16	17	18
4	19	20	21	22	23	24
5	25	26	27	28	29	30
6	31	32	33	34	35	36
7	37	38	39	40	41	42
8	43	44	45	46	47	48
9	49	50	51	52	53	54
10	55	56	57	58	59	60
11	61	62	63	64	65	66
12	67	68	69	70	71	72
13	73	74	75	76	77	78
14	79	80	81	82	83	84
15	85	86	87	88	89	90
16	91	92	93	94	95	96
17	97	98	99	100	101	102
18	103	104	105	106	107	108
19	109	110	111	112	113	114
20	115	116	117	118	119	120
21	121	122	123	124	125	126
22	127	128	129	130	131	132
23	133	134	135	136	137	138
24	139	140	141	142	143	144
25	145	146	147	148	149	150
26	151	152	153	154	155	156
27	157	158	159	160	161	162
28	163	164	165	166	167	168
29	169	170	171	172	173	174
30	175	176	177	178	179	180
31	181	182	183	184	185	186
32	187	188	189	190	191	192
33	193	194	195	196	197	198
34	199	200	201	202	203	204
35	205	206	207	208	209	210
36	211	212	213	214	215	216

37	217	218	219	220	221	222
38	223	224	225	226	227	228

1S P R I N G   E L E M E N T S

ELEMENT TYPE = 1
NUMBER OF ELEMENTS = 30

ELEMENT LOAD CASE MULTIPLIERS

CASE (A)	CASE (B)	CASE (C)	CASE (D)
.0000	.0000	.0000	.0000

ELEMENT NUMBER	NODE (N)	SUPPORT GROUP	CODE KD	CODE KR	DIRECTION COSINES WRT GLOBAL AXES X-	Y-	Z-	SPECIFIED DISPLACEMENT	SPECIFIED ROTATION	SPRING RATE
1	1	1	1	0	1.000	.000	.000	.000	.000	1.0000E+11
2	1	1	1	0	.000	1.000	.000	.000	.000	1.0000E+11
3	1	1	1	0	.000	.000	1.000	.000	.000	1.0000E+11
4	1	1	0	1	1.000	.000	.000	.000	.000	1.0000E+20
5	1	1	0	1	.000	1.000	.000	.000	.000	1.0000E+20
6	1	1	0	1	.000	.000	1.000	.000	.000	1.0000E+20
7	4	1	1	0	1.000	.000	.000	.000	.000	1.0000E+08
8	4	1	1	0	.000	.000	1.000	.000	.000	1.0000E+08
9	7	1	1	0	.000	1.000	.000	.000	.000	1.0000E+08
10	11	1	1	0	.000	1.000	.000	.000	.000	1.0000E+08
11	11	1	1	0	.000	.000	1.000	.000	.000	1.0000E+05
12	15	1	1	0	1.000	.000	.000	.000	.000	1.0000E+05
13	17	1	1	0	.000	1.000	.000	.000	.000	1.0000E+08
14	17	1	1	0	.000	.000	1.000	.000	.000	1.0000E+05
15	36	2	1	0	.000	1.000	.000	.000	.000	1.0000E+08
16	36	2	1	0	.000	.000	1.000	.000	.000	1.0000E+05
17	38	2	1	0	1.000	.000	.000	.000	.000	1.0000E+11
18	38	2	1	0	.000	1.000	.000	.000	.000	1.0000E+11
19	38	2	1	0	.000	.000	1.000	.000	.000	1.0000E+11
20	38	2	0	1	1.000	.000	.000	.000	.000	1.0000E+20
21	38	2	0	1	.000	1.000	.000	.000	.000	1.0000E+20
22	38	2	0	1	.000	.000	1.000	.000	.000	1.0000E+20
23	23	2	1	0	1.000	.000	.000	.000	.000	1.0000E+05
24	23	2	1	0	.000	1.000	.000	.000	.000	1.0000E+08
25	31	2	1	0	1.000	.000	.000	.000	.000	1.0000E+11
26	31	2	1	0	.000	1.000	.000	.000	.000	1.0000E+11
27	31	2	1	0	.000	.000	1.000	.000	.000	1.0000E+11
28	31	2	0	1	1.000	.000	.000	.000	.000	1.0000E+20
29	31	2	0	1	.000	1.000	.000	.000	.000	1.0000E+20
30	31	2	0	1	.000	.000	1.000	.000	.000	1.0000E+20

1P I P E   E L E M E N T   I N P U T   D A T A

C O N T R O L   I N F O R M A T I O N

NUMBER OF PIPE ELEMENTS = 37

NUMBER OF MATERIAL SETS = 1

MAXIMUM NUMBER OF MATERIAL TEMPERATURE INPUT POINTS	=	1
NUMBER OF SECTION PROPERTY SETS	=	3
NUMBER OF BRANCH POINT NODES	=	0
MAXIMUM NUMBER OF TANGENTS COMMON TO A BRANCH POINT	=	3
FLAG FOR NEGLECTING AXIAL DEFORMATIONS IN BEND ELEMENTS (EQ.1, NEGLECT)	=	0

1M A T E R I A L   P R O P E R T Y   T A B L E S

0MATERIAL NUMBER = (1)
NUMBER OF TEMPERATURE POINTS = (1)
IDENTIFICATION = ( )

POINT NUMBER	TEMPERATURE	YOUNG'S MODULUS	POISSON'S RATIO	THERMAL EXPANSION
1	.00	2.900E+07	.300	6.440E-06

1S E C T I O N   P R O P E R T Y   T A B L E

SECTION NUMBER	OUTSIDE DIAMETER	WALL THICKNESS	SHAPE FACTOR FOR SHEAR	WEIGHT/ UNIT LENGTH	MASS/ UNIT LENGTH	D E S C R I P T I O N
1	3.500	.2160	.0000	8.9800E-01	2.3240E-03	
2	4.500	.2370	.0000	1.3580E+00	3.5145E-03	
3	8.625	.3220	.0000	4.1870E+00	1.0836E-02	

E L E M E N T   L O A D   C A S E   M U L T I P L I E R S

	CASE A	CASE B	CASE C	CASE D
X-DIRECTION GRAVITY	.000	.000	.000	.000
Y-DIRECTION GRAVITY	.000	.000	.000	.000
Z-DIRECTION GRAVITY	.000	.000	.000	.000
THERMAL DISTORTION	.000	.000	.000	.000
PRESSURE DISTORTION	.000	.000	.000	.000

1P I P E   E L E M E N T   I N P U T   D A T A

ELEMENT NUMBER	ELEMENT TYPE	NODE -I	NODE -J	MATL. NUMBER	SECTION NUMBER	REFERENCE TEMPERATURE (BEND RADIUS)	DESIGN PRESSURE (THIRD POINT)	PEAK PRESSURE (X3-ORDINATE)	TEST PRESSURE (Y3-ORDINATE)	END CODES END-I	END CODES END-J (Z3-ORDINATE)	NODE INCREMENT (BEND DEGREE)	INPUT TAG
1	TANGENT	1	2	1	1	50.00	200.00	.00	.00	0	0	1	II
2	BEND	2	3	1	1	50.00	200.00	.00	.00	0	0		IC
						(4.500)	(TI)	(19.500)	(.000)		(.000)	(90.0001)	

3	TANGENT	3	4	1	1	50.00	200.00	.00	.00	0	0	1	II
4	TANGENT	4	5	1	1	50.00	200.00	.00	.00	0	0	1	II
5	BEND	5	6	1	1	50.00	200.00	.00	.00	0	0		IC
						(4.500)	(TI)	(19.500)	(-204.000)	(.000)		(90.0001)	
6	TANGENT	6	7	1	1	50.00	200.00	.00	.00	0	0	1	II
7	BEND	7	8	1	1	50.00	200.00	.00	.00	0	0		IC
						(4.500)	(TI)	(19.500)	(-204.000)	(144.000)		(90.0001)	
8	TANGENT	8	9	1	1	50.00	200.00	.00	.00	0	0	1	II
9	TANGENT	9	10	1	1	50.00	200.00	.00	.00	0	0	1	II
10	TANGENT	10	11	1	1	50.00	200.00	.00	.00	0	0	1	II
11	TANGENT	11	12	1	1	50.00	200.00	.00	.00	0	0	1	II
12	TANGENT	12	13	1	1	50.00	200.00	.00	.00	0	0	1	II
13	BEND	13	14	1	1	50.00	200.00	.00	.00	0	0		IC
						(4.500)	(TI)	(487.500)	(-204.000)	(144.000)		(90.0001)	
14	TANGENT	14	15	1	1	50.00	200.00	.00	.00	0	0	1	II
15	TANGENT	15	16	1	1	50.00	200.00	.00	.00	0	0	1	II
16	BEND	16	17	1	1	50.00	200.00	.00	.00	0	0		IC
						(4.500)	(TI)	(487.500)	(-204.000)	(240.000)		(90.0001)	
17	TANGENT	17	18	1	1	50.00	200.00	.00	.00	0	0	1	II
18	TANGENT	18	19	1	1	50.00	200.00	.00	.00	0	0	1	II
19	BEND	19	20	1	1	50.00	200.00	.00	.00	0	0		IC
						(4.500)	(TI)	(727.500)	(-204.000)	(240.000)		(90.0001)	
20	TANGENT	20	21	1	1	50.00	200.00	.00	.00	0	0	1	II
21	TANGENT	21	22	1	2	50.00	200.00	.00	.00	0	0	1	II
22	TANGENT	22	23	1	2	50.00	200.00	.00	.00	0	0	1	II
23	BEND	23	24	1	2	50.00	200.00	.00	.00	0	0		IC
						(6.000)	(TI)	(727.500)	(-264.000)	(184.000)		(90.0001)	
24	TANGENT	24	25	1	2	50.00	200.00	.00	.00	0	0	1	II
25	TANGENT	25	26	1	2	50.00	200.00	.00	.00	0	0	1	II
26	BEND	26	27	1	2	50.00	200.00	.00	.00	0	0		IC
						(6.000)	(TI)	(851.500)	(-264.000)	(184.000)		(90.0001)	
27	TANGENT	27	28	1	2	50.00	200.00	.00	.00	0	0	1	II
28	TANGENT	28	29	1	2	50.00	200.00	.00	.00	0	0	1	II
29	BEND	29	30	1	2	50.00	200.00	.00	.00	0	0		IC
						(6.000)	(TI)	(851.500)	(-264.000)	(136.000)		(90.0001)	
30	TANGENT	30	31	1	2	50.00	200.00	.00	.00	0	0	1	II
31	TANGENT	21	32	1	3	50.00	200.00	.00	.00	0	0	1	II
32	TANGENT	32	33	1	3	50.00	200.00	.00	.00	0	0	1	II

1P I P E   E L E M E N T   I N P U T   D A T A

ELEMENT NUMBER	ELEMENT TYPE	NODE -I	NODE -J	MATL. NUMBER	SECTION NUMBER	REFERENCE TEMPERATURE (BEND RADIUS)	DESIGN PRESSURE (THIRD POINT)	PEAK PRESSURE (X3-ORDINATE)	TEST PRESSURE (Y3-ORDINATE)	END CODES END-I (Z3-ORDINATE)	END-J	NODE INCREMENT (BEND DEGREE)	INPUT TAG
33	TANGENT	33	34	1	3	50.00	200.00	.00	.00	0	0	1	II
34	TANGENT	34	35	1	3	50.00	200.00	.00	.00	0	0	1	II
35	BEND	35	36	1	3	50.00	200.00	.00	.00	0	0		IC
						(12.000)	(TI)	(727.500)	(-264.000)	(426.000)		(90.0001)	
36	TANGENT	36	37	1	3	50.00	200.00	.00	.00	0	0	1	II

37 TANGENT 37 38 1 3 50.00 200.00 .00 .00 0 0 1 II

1B A N D W I D T H M I N I M I Z A T I O N

MINBND (BANDWIDTH CONTROL PARAMETER)= 1

0EQUATION NUMBERS AFTER BANDWIDTH MINIMIZATION

0NODE	SEQUENCE	X	Y	Z	XX	YY	ZZ
1	38	223	224	225	226	227	228
2	37	217	218	219	220	221	222
3	36	211	212	213	214	215	216
4	35	205	206	207	208	209	210
5	34	199	200	201	202	203	204
6	33	193	194	195	196	197	198
7	32	187	188	189	190	191	192
8	31	181	182	183	184	185	186
9	30	175	176	177	178	179	180
10	29	169	170	171	172	173	174
11	28	163	164	165	166	167	168
12	27	157	158	159	160	161	162
13	26	151	152	153	154	155	156
14	25	145	146	147	148	149	150
15	24	139	140	141	142	143	144
16	23	133	134	135	136	137	138
17	22	127	128	129	130	131	132
18	21	121	122	123	124	125	126
19	20	115	116	117	118	119	120
20	19	109	110	111	112	113	114
21	18	103	104	105	106	107	108
22	17	97	98	99	100	101	102
23	15	85	86	87	88	89	90
24	13	73	74	75	76	77	78
25	11	61	62	63	64	65	66
26	9	49	50	51	52	53	54
27	7	37	38	39	40	41	42
28	5	25	26	27	28	29	30
29	3	13	14	15	16	17	18
30	2	7	8	9	10	11	12
31	1	1	2	3	4	5	6
32	16	91	92	93	94	95	96
33	14	79	80	81	82	83	84
34	12	67	68	69	70	71	72
35	10	55	56	57	58	59	60
36	8	43	44	45	46	47	48
37	6	31	32	33	34	35	36
38	4	19	20	21	22	23	24

0BANDWIDTH PRIOR TO RESEQUENCING= 72

BANDWIDTH AFTER RESEQUENCING = 18

1E Q U A T I O N P A R A M E T E R S

TOTAL NUMBER OF EQUATIONS = 228

BANDWIDTH = 18

NUMBER OF EQUATIONS IN A BLOCK = 228

NUMBER OF BLOCKS = 1

1N O D A L L O A D S (S T A T I C) O R M A S S E S (D Y N A M I C)

NODE NUMBER	LOAD CASE	X-AXIS FORCE	Y-AXIS FORCE	Z-AXIS FORCE	X-AXIS MOMENT	Y-AXIS MOMENT	Z-AXIS MOMENT

1D Y N A M I C A N A L Y S I S

Nodal Masses of the Model BM3

Unit:lbs/in/sec**2

Node No.	Mass
1	0.017
2	0.026
3	0.212
4	0.227
5	0.031
6	0.165
7	0.165
8	0.092
9	0.267
10	0.275
11	0.182
12	0.174
13	0.092
14	0.059
15	0.101
16	0.059
17	0.105
18	0.268
19	0.18
20	0.073
21	0.207
22	0.088
23	0.043
24	0.052
25	0.197
26	0.178
27	0.048
28	0.063
29	0.048
30	0.175
31	0.158
32	0.163
33	0.276
34	0.78
35	0.687
36	0.687
37	1.17
38	0.585

MODE NUMBER		CIRCULAR FREQUENCY (RAD/SEC)	FREQUENCY (CYCLES/SEC)	PERIOD (SEC)
0	1	1.8268E+01	2.9074E+00	3.4395E-01
0	2	2.7565E+01	4.3871E+00	2.2794E-01
0	3	3.4657E+01	5.5158E+00	1.8130E-01
0	4	3.5840E+01	5.7041E+00	1.7531E-01
0	5	4.3841E+01	6.9775E+00	1.4332E-01
0	6	4.6141E+01	7.3436E+00	1.3617E-01
0	7	4.9509E+01	7.8796E+00	1.2691E-01
0	8	6.4727E+01	1.0302E+01	9.7071E-02
0	9	6.9466E+01	1.1056E+01	9.0450E-02
0	10	7.0578E+01	1.1233E+01	8.9025E-02
0	11	7.2241E+01	1.1498E+01	8.6975E-02
0	12	7.8104E+01	1.2431E+01	8.0446E-02
0	13	8.7205E+01	1.3879E+01	7.2050E-02
0	14	1.0130E+02	1.6122E+01	6.2026E-02
0	15	1.1333E+02	1.8038E+01	5.5439E-02
0	16	1.1700E+02	1.8621E+01	5.3702E-02
0	17	1.2265E+02	1.9521E+01	5.1227E-02
0	18	1.2291E+02	1.9561E+01	5.1122E-02
0	19	1.3707E+02	2.1815E+01	4.5840E-02
0	20	1.3935E+02	2.2179E+01	4.5088E-02
0	21	1.4361E+02	2.2857E+01	4.3750E-02
0	22	1.6305E+02	2.5950E+01	3.8536E-02
0	23	2.4517E+02	3.9020E+01	2.5628E-02
0	24	2.4831E+02	3.9521E+01	2.5303E-02
0	25	2.5890E+02	4.1205E+01	2.4269E-02
0	26	2.8845E+02	4.5908E+01	2.1783E-02
0	27	2.9688E+02	4.7249E+01	2.1164E-02
0	28	3.3072E+02	5.2636E+01	1.8999E-02
0	29	3.7253E+02	5.9290E+01	1.6866E-02
0	30	4.4039E+02	7.0090E+01	1.4267E-02
0	31	4.4421E+02	7.0699E+01	1.4144E-02

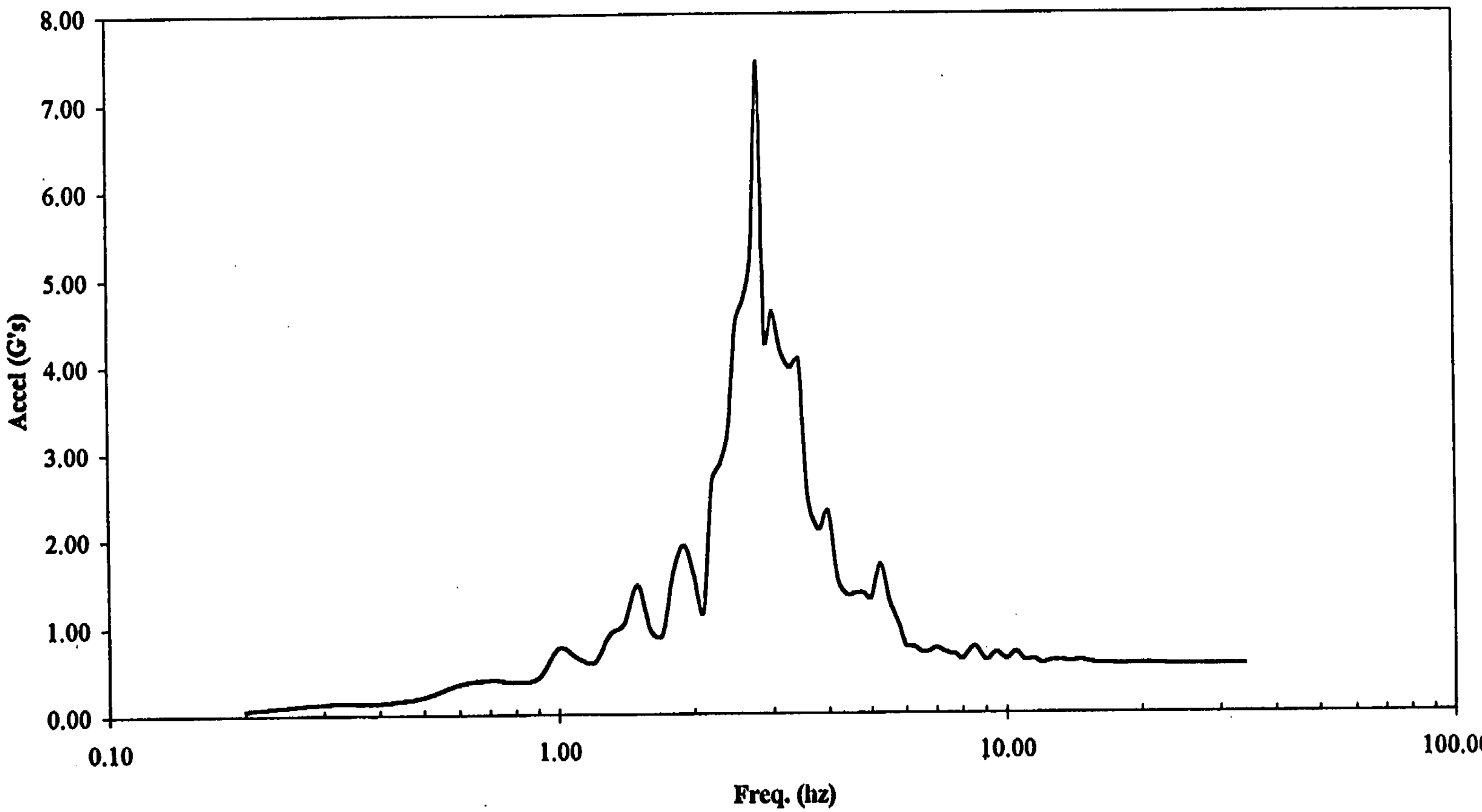
1MODAL PARTICIPATION FACTORS

MODE	FREQ(CPS)	X-DIRECTION	Y-DIRECTION	Z-DIRECTION
1	2.907	-2.3473E-02	2.7695E-01	3.5437E-02
2	4.387	-5.5673E-01	9.0279E-02	-1.0926E-01
3	5.516	-4.7719E-02	-9.4682E-01	-1.9203E-01
4	5.704	-9.9555E-03	-2.1351E-01	6.8419E-01
5	6.977	-1.5342E-01	-4.5827E-01	7.4423E-01
6	7.344	5.5364E-01	3.9813E-01	-3.5022E-01
7	7.880	-3.7119E-01	3.9334E-01	4.5914E-01
8	10.302	-4.0272E-01	1.1940E+00	-1.1466E-02
9	11.056	5.5995E-01	1.8119E-01	-7.0772E-02
10	11.233	9.2828E-02	-1.4703E-01	-2.7472E-04
11	11.498	-1.4351E+00	-1.9653E-01	-3.4271E-01
12	12.431	4.2956E-01	-2.4824E-01	-4.5713E-01
13	13.879	-3.2298E-01	-1.5196E-01	-5.3436E-02
14	16.122	9.7048E-02	4.1157E-01	2.9944E-01
15	18.038	-1.0796E-01	4.0371E-02	4.3020E-01
16	18.621	-4.2217E-01	-4.0211E-02	2.9074E-01
17	19.521	-3.2449E-02	-2.8297E-01	-2.1127E-01
18	19.561	-1.4330E-01	8.6254E-02	-6.1027E-01
19	21.815	-3.8758E-01	1.1586E-01	-5.3784E-02
20	22.179	-3.6788E-03	-9.8705E-01	7.5035E-02
21	22.857	-6.9162E-02	8.0846E-02	1.1833E+00
22	25.950	1.6498E-01	-1.8727E-02	1.6100E+00
23	39.020	1.2766E-01	7.3949E-01	-2.9101E-02
24	39.521	-2.4041E-02	-3.3916E-01	-6.4757E-03
25	41.205	-2.0186E-02	3.3298E-01	-1.9897E-02
26	45.908	4.7943E-01	-1.6665E-02	-6.2062E-02
27	47.249	-8.3761E-02	-2.6544E-01	2.6515E-02
28	52.636	-3.3621E-01	4.7051E-02	1.1705E-01

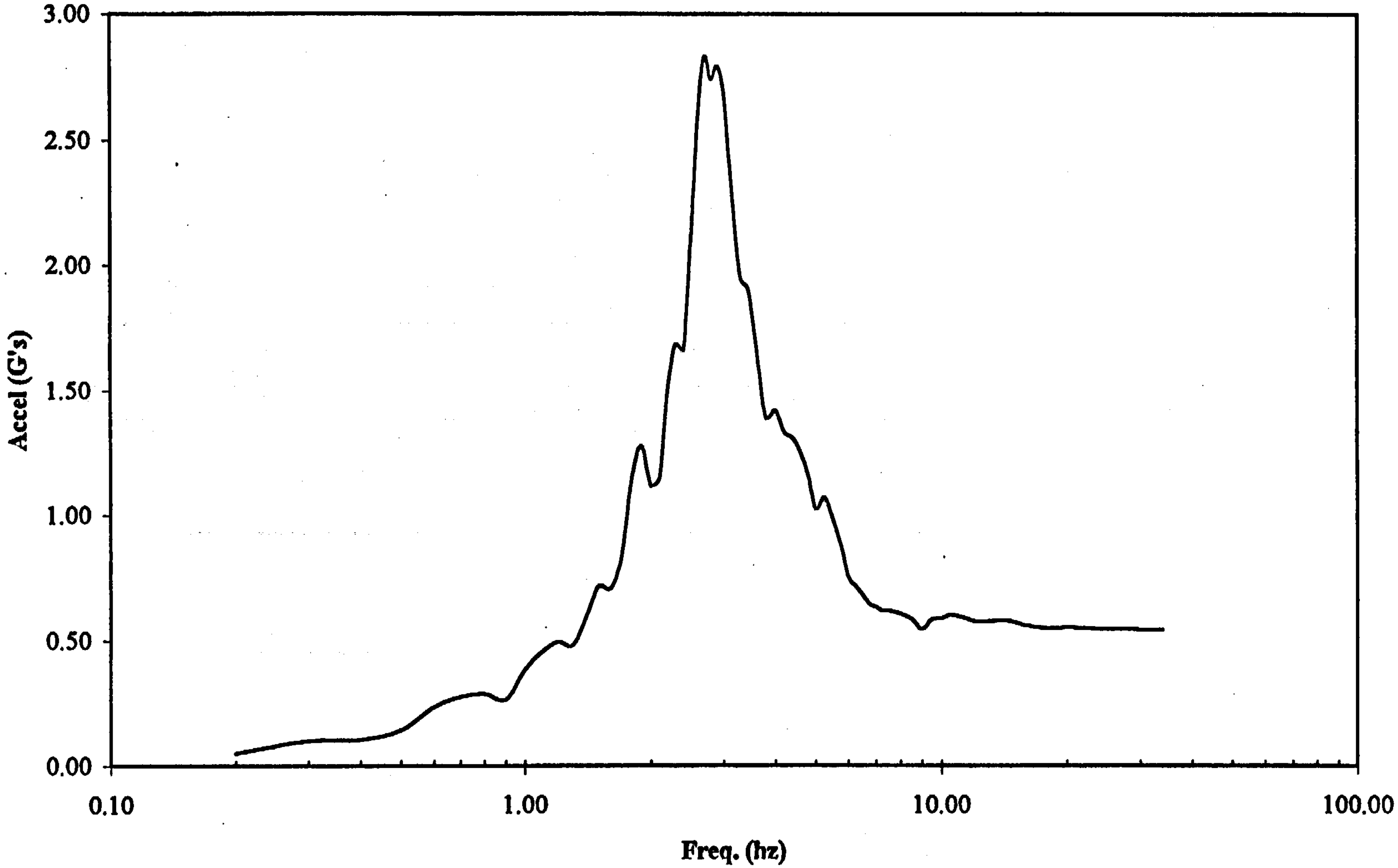
29	59.290	-2.5127E-03	2.4614E-04	-5.0636E-01
30	70.090	4.5397E-02	1.7356E-03	-6.2325E-01
31	70.699	-5.3524E-01	-1.2430E-02	-6.2982E-02

APPENDIX B

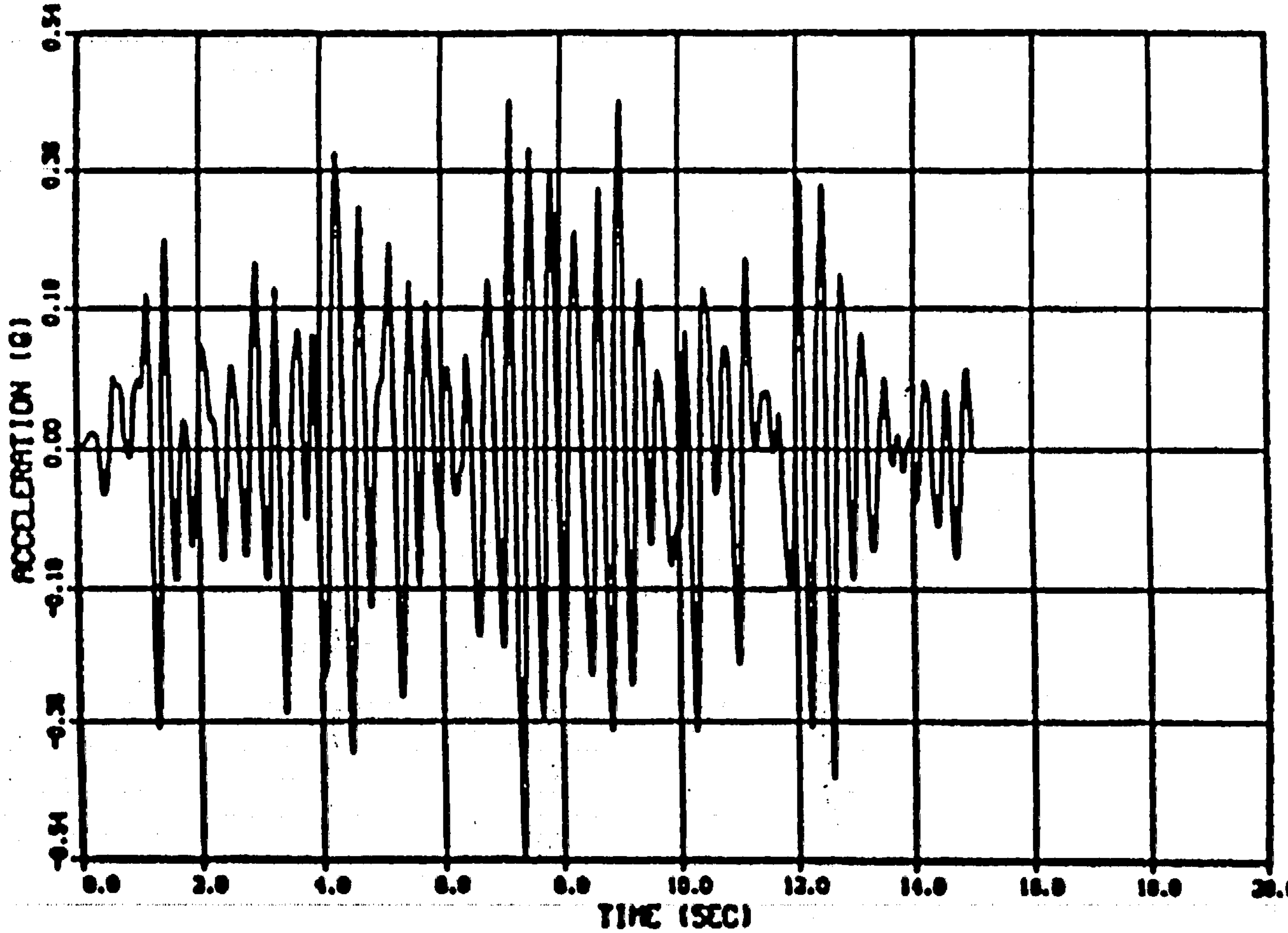
TIME HISTORY AND RESPONSE SPECTRUM INPUT FOR BM3 ANALYSIS



BM3 Input Spectrum (1%)



BM3 Input Spectrum (5%)



BM3 TIME HISTORY RECORD

```
24T   24   25   1   2
25T   25   26   1   2
26B   26   27   1   2
  6.0      TI   851.5     -264.0    184.0
27T   27   28   1   2
28T   28   29   1   2
29B   29   30   1   2
  6.0      TI   851.5     -264.0    136.0
30T   30   31   1   2
31T   21   32   1   3
32T   32   33   1   3
33T   33   34   1   3
34T   34   35   1   3
35B   35   36   1   3
  12.0     TI   727.5     -264.0    426.0
36T   36   37   1   3
37T   37   38   1   3

                                          TIME HISTORY ANALYSIS
                               100.0
  1    1    14950   20         0.001      0.01

  1

1500      1.0 OPERATIONAL LEVEL
   .000    .025     .020    .063     .030    .127     .040    .247     .050    .436
   .060    .703     .070   1.060     .080   1.488     .090   1.933     .100   2.343
   .110   2.707     .120   3.036     .130   3.341     .140   3.653     .150   4.016
   .160   4.481     .170   5.053     .180   5.720     .190   6.460     .200   7.199
   .210   7.839     .220   8.272     .230   8.433     .240   8.349     .250   8.119
   .260   7.898     .270   7.763     .280   7.730     .290   7.780     .300   7.760
   .310   7.422     .320   6.549     .330   4.924     .340   2.590     .350   -.200
   .360  -3.220     .370  -6.119     .380  -8.900     .390 -12.090     .400 -15.814
   .410 -19.350     .420 -21.900     .430 -23.220     .440 -23.510     .450 -23.130
   .460 -22.220     .470 -20.680     .480 -18.410     .490 -15.490     .500 -12.170
   .510  -8.580     .520  -4.550     .530    .050     .540   5.380     .550  11.680
   .560  18.610     .570  25.200     .580  30.370     .590  33.830     .600  35.640
   .610  35.460     .620  33.590     .630  31.710     .640  31.080     .650  31.166
   .660  31.170     .670  31.120     .680  31.290     .690  31.390     .700  30.820
   .710  29.340     .720  26.646     .730  22.460     .740  17.180     .750  11.540
   .760   6.556     .770   3.236     .780   1.835     .790   1.780     .800   2.030
   .810   1.531     .820   -.130     .830  -2.250     .840  -3.930     .850  -5.020
   .860  -5.220     .870  -3.630     .880    .330     .890   6.127     .900  12.550
   .910  18.613     .920  23.618     .930  26.986     .940  28.850     .950  30.020
   .960  31.190     .970  32.500     .980  33.750     .990  34.540    1.000  34.560
  1.010  33.949    1.020  32.890    1.030  31.460    1.040  30.040    1.050  29.340
  1.060  30.180    1.070  33.240    1.080  38.420    1.090  44.810    1.100  51.475
  1.110  57.912    1.120  64.040    1.130  69.550    1.140  73.800    1.150  76.150
  1.160  75.680    1.170  70.900    1.180  60.900    1.190  46.110    1.200  27.670
  1.210   6.520    1.220 -16.180    1.230 -39.133    1.240 -61.080    1.250 -81.110
  1.260 -98.830    1.270-113.940    1.280-126.200    1.290-135.900    1.300-142.200
  1.310-143.250    1.320-137.700    1.330-126.150    1.340-109.900    1.350 -90.300
  1.360 -68.300    1.370 -45.000    1.380 -21.051    1.390   3.690    1.400  28.670
  1.410  52.250    1.420  72.600    1.430  88.450    1.440  99.200    1.450 104.800
  1.460 105.300    1.470 100.650    1.480  91.100    1.490  77.830    1.500  62.140
  1.510  45.060    1.520  27.540    1.530  10.580    1.540  -5.064    1.550 -18.940
  1.560 -31.120    1.570 -41.830    1.580 -51.170    1.590 -58.910    1.600 -64.200
  1.610 -66.070    1.620 -63.940    1.630 -58.020    1.640 -49.480    1.650 -39.460
  1.660 -29.300    1.670 -20.800    1.680 -14.520    1.690  -9.130    1.700  -3.370
```

1.710	2.640	1.720	7.910	1.730	11.692	1.740	13.840	1.750	14.260
1.760	13.490	1.770	12.720	1.780	12.340	1.790	11.590	1.800	9.390
1.810	5.210	1.820	-1.060	1.830	-9.275	1.840	-18.870	1.850	-28.840
1.860	-37.870	1.870	-44.660	1.880	-48.440	1.890	-49.200	1.900	-47.540
1.910	-44.430	1.920	-40.640	1.930	-36.040	1.940	-30.680	1.950	-25.310
1.960	-20.220	1.970	-14.660	1.980	-7.530	1.990	1.474	2.000	11.810
2.010	22.690	2.020	32.910	2.030	41.160	2.040	46.760	2.050	49.810
2.060	50.900	2.070	50.890	2.080	50.370	2.090	49.540	2.100	48.330
2.110	46.500	2.120	43.980	2.130	40.770	2.140	36.980	2.150	33.160
2.160	29.580	2.170	25.820	2.180	21.754	2.190	18.100	2.200	15.660
2.210	14.300	2.220	13.650	2.230	13.840	2.240	14.633	2.250	15.120
2.260	14.350	2.270	11.800	2.280	7.500	2.290	2.070	2.300	-3.854
2.310	-10.127	2.320	-16.677	2.330	-23.257	2.340	-29.545	2.350	-35.200
2.360	-40.270	2.370	-45.040	2.380	-49.500	2.390	-53.310	2.400	-55.620
2.410	-55.050	2.420	-51.130	2.430	-44.950	2.440	-37.650	2.450	-29.410
2.460	-20.100	2.470	-9.890	2.480	.560	2.490	10.110	2.500	18.160
2.510	25.070	2.520	31.250	2.530	36.340	2.540	39.640	2.550	40.830
2.560	40.210	2.570	38.300	2.580	35.700	2.590	32.830	2.600	29.700
2.610	26.130	2.620	22.040	2.630	17.430	2.640	12.430	2.650	7.293
2.660	2.293	2.670	-2.550	2.680	-7.200	2.690	-11.440	2.700	-15.455
2.710	-19.818	2.720	-25.084	2.730	-31.340	2.740	-38.050	2.750	-44.450
2.760	-49.700	2.770	-52.860	2.780	-53.530	2.790	-52.120	2.800	-48.890
2.810	-43.630	2.820	-35.990	2.830	-25.760	2.840	-13.340	2.850	.310
2.860	14.222	2.870	27.998	2.880	41.354	2.890	53.810	2.900	64.880
2.910	74.050	2.920	81.260	2.930	86.860	2.940	90.700	2.950	92.200
2.960	90.090	2.970	83.490	2.980	72.430	2.990	58.100	3.000	42.290
3.010	26.590	3.020	12.040	3.030	-.760	3.040	-11.720	3.050	-21.500
3.060	-30.694	3.070	-39.100	3.080	-46.480	3.090	-53.110	3.100	-59.020
3.110	-63.370	3.120	-65.550	3.130	-65.650	3.140	-63.900	3.150	-60.550
3.160	-55.090	3.170	-46.680	3.180	-34.860	3.190	-19.850	3.200	-2.720
3.210	14.938	3.220	31.850	3.230	47.100	3.240	60.000	3.250	70.250
3.260	77.000	3.270	79.050	3.280	75.700	3.290	67.100	3.300	54.100
3.310	38.090	3.320	20.270	3.330	.830	3.340	-19.900	3.350	-41.140
3.360	-61.870	3.370	-80.710	3.380	-97.000	3.390	-111.450	3.400	-123.830
3.410	-132.440	3.420	-135.540	3.430	-132.490	3.440	-124.440	3.450	-113.420
3.460	-101.280	3.470	-88.960	3.480	-76.420	3.490	-63.210	3.500	-49.130
3.510	-34.416	3.520	-19.460	3.530	-4.590	3.540	9.570	3.550	21.940
3.560	31.600	3.570	38.390	3.580	42.880	3.590	46.040	3.600	48.730
3.610	51.340	3.620	53.910	3.630	56.150	3.640	57.860	3.650	58.800
3.660	58.620	3.670	56.650	3.680	52.200	3.690	45.550	3.700	36.660
3.710	25.204	3.720	11.500	3.730	-3.020	3.740	-16.340	3.750	-26.690
3.760	-33.270	3.770	-36.140	3.780	-35.800	3.790	-32.590	3.800	-26.660
3.810	-17.920	3.820	-6.674	3.830	6.614	3.840	20.760	3.850	33.940
3.860	44.630	3.870	52.060	3.880	55.970	3.890	56.520	3.900	53.760
3.910	47.570	3.920	38.150	3.930	26.060	3.940	12.130	3.950	-2.690
3.960	-17.930	3.970	-33.551	3.980	-49.341	3.990	-64.670	4.000	-78.600
4.010	-90.240	4.020	-99.270	4.030	-105.970	4.040	-110.800	4.050	-113.900
4.060	-115.210	4.070	-114.590	4.080	-111.900	4.090	-106.970	4.100	-99.680
4.110	-89.450	4.120	-76.210	4.130	-60.730	4.140	-43.930	4.150	-26.260
4.160	-7.650	4.170	12.124	4.180	33.040	4.190	54.610	4.200	75.930
4.210	95.900	4.220	113.500	4.230	128.010	4.240	138.790	4.250	145.480
4.260	148.210	4.270	147.310	4.280	143.810	4.290	138.900	4.300	133.100
4.310	126.400	4.320	117.580	4.330	105.240	4.340	88.790	4.350	68.820
4.360	46.740	4.370	24.450	4.380	3.020	4.390	-17.985	4.400	-38.970
4.410	-59.390	4.420	-78.560	4.430	-96.210	4.440	-112.440	4.450	-127.390
4.460	-140.460	4.470	-150.200	4.480	-155.300	4.490	-155.600	4.500	-151.300
4.510	-143.250	4.520	-131.860	4.530	-116.610	4.540	-97.660	4.550	-76.090
4.560	-53.190	4.570	-29.880	4.580	-6.244	4.590	18.100	4.600	42.890
4.610	66.470	4.620	86.900	4.630	103.150	4.640	114.700	4.650	120.700
4.660	120.700	4.670	115.300	4.680	105.710	4.690	93.570	4.700	80.350

4.710	66.900	4.720	53.190	4.730	38.710	4.740	22.900	4.750	5.790
4.760	-12.130	4.770	-30.350	4.780	-47.780	4.790	-62.290	4.800	-72.300
4.810	-77.800	4.820	-79.400	4.830	-77.130	4.840	-70.940	4.850	-61.060
4.860	-48.520	4.870	-34.956	4.880	-22.100	4.890	-10.920	4.900	-1.644
4.910	5.828	4.920	11.809	4.930	16.510	4.940	20.270	4.950	23.770
4.960	27.212	4.970	29.890	4.980	31.140	4.990	31.300	5.000	31.340
5.010	31.767	5.020	33.100	5.030	36.332	5.040	41.640	5.050	48.030
5.060	54.406	5.070	60.560	5.080	66.810	5.090	72.899	5.100	78.840
5.110	85.550	5.120	92.980	5.130	99.230	5.140	102.200	5.150	101.160
5.160	96.100	5.170	87.260	5.180	75.150	5.190	60.370	5.200	43.860
5.210	26.840	5.220	10.090	5.230	-6.380	5.240	-23.250	5.250	-41.154
5.260	-59.950	5.270	-78.380	5.280	-94.900	5.290	-108.610	5.300	-118.910
5.310	-125.050	5.320	-126.600	5.330	-124.140	5.340	-118.560	5.350	-110.380
5.360	-100.000	5.370	-88.120	5.380	-74.640	5.390	-58.930	5.400	-40.500
5.410	-19.920	5.420	1.700	5.430	23.380	5.440	43.670	5.450	60.900
5.460	73.590	5.470	80.700	5.480	82.600	5.490	80.500	5.500	75.990
5.510	69.310	5.520	60.290	5.530	48.950	5.540	35.770	5.550	21.310
5.560	6.090	5.570	-9.037	5.580	-23.540	5.590	-37.540	5.600	-50.530
5.610	-60.550	5.620	-66.000	5.630	-67.350	5.640	-65.170	5.650	-58.730
5.660	-47.990	5.670	-34.340	5.680	-19.900	5.690	-6.430	5.700	5.590
5.710	16.663	5.720	27.730	5.730	39.400	5.740	51.120	5.750	61.180
5.760	68.200	5.770	72.050	5.780	72.950	5.790	70.580	5.800	64.910
5.810	56.900	5.820	48.100	5.830	39.740	5.840	32.480	5.850	26.270
5.860	20.580	5.870	14.830	5.880	8.700	5.890	2.164	5.900	-4.749
5.910	-12.073	5.920	-19.580	5.930	-26.450	5.940	-32.030	5.950	-36.390
5.960	-39.620	5.970	-41.330	5.980	-40.840	5.990	-37.730	6.000	-31.830
6.010	-23.420	6.020	-13.170	6.030	-1.850	6.040	9.490	6.050	19.690
6.060	27.790	6.070	33.310	6.080	36.530	6.090	38.280	6.100	39.370
6.110	40.050	6.120	40.120	6.130	39.100	6.140	36.540	6.150	32.410
6.160	26.630	6.170	19.130	6.180	10.267	6.190	.742	6.200	-8.230
6.210	-15.060	6.220	-19.150	6.230	-21.340	6.240	-22.740	6.250	-23.650
6.260	-23.810	6.270	-23.000	6.280	-21.259	6.290	-18.869	6.300	-16.310
6.310	-13.879	6.320	-11.870	6.330	-10.620	6.340	-10.140	6.350	-9.950
6.360	-8.950	6.370	-5.970	6.380	-.301	6.390	8.060	6.400	18.110
6.410	28.230	6.420	36.780	6.430	42.760	6.440	45.750	6.450	45.640
6.460	42.950	6.470	38.750	6.480	34.000	6.490	28.850	6.500	22.870
6.510	15.470	6.520	6.080	6.530	-5.390	6.540	-18.610	6.550	-32.987
6.560	-47.530	6.570	-61.120	6.580	-72.690	6.590	-81.420	6.600	-87.220
6.610	-90.970	6.620	-93.460	6.630	-94.480	6.640	-93.640	6.650	-91.160
6.660	-87.140	6.670	-80.840	6.680	-71.600	6.690	-59.680	6.700	-45.690
6.710	-29.680	6.720	-12.126	6.730	5.750	6.740	22.550	6.750	37.410
6.760	49.890	6.770	59.850	6.780	67.540	6.790	73.550	6.800	78.250
6.810	81.660	6.820	83.550	6.830	83.470	6.840	81.160	6.850	76.910
6.860	70.890	6.870	62.720	6.880	52.170	6.890	39.700	6.900	26.120
6.910	12.040	6.920	-1.990	6.930	-15.190	6.940	-27.070	6.950	-37.960
6.960	-48.116	6.970	-57.360	6.980	-65.737	6.990	-73.330	7.000	-80.510
7.010	-87.850	7.020	-94.880	7.030	-99.600	7.040	-100.100	7.050	-95.350
7.060	-85.640	7.070	-71.000	7.080	-51.800	7.090	-29.710	7.100	-5.780
7.110	19.780	7.120	46.806	7.130	74.450	7.140	101.430	7.150	126.350
7.160	147.900	7.170	163.750	7.180	172.600	7.190	174.250	7.200	168.300
7.210	154.150	7.220	131.300	7.230	101.150	7.240	65.960	7.250	28.530
7.260	-9.040	7.270	-45.377	7.280	-79.830	7.290	-112.040	7.300	-141.300
7.310	-166.450	7.320	-186.400	7.330	-200.200	7.340	-207.200	7.350	-206.900
7.360	-199.400	7.370	-184.800	7.380	-163.900	7.390	-138.200	7.400	-109.200
7.410	-78.470	7.420	-47.140	7.430	-15.687	7.440	15.267	7.450	44.560
7.460	71.130	7.470	94.400	7.480	114.000	7.490	129.900	7.500	141.500
7.510	148.400	7.520	150.100	7.530	146.250	7.540	137.200	7.550	124.300
7.560	108.300	7.570	89.500	7.580	67.410	7.590	42.110	7.600	14.440
7.610	-13.927	7.620	-41.410	7.630	-67.080	7.640	-90.270	7.650	-110.000
7.660	-125.100	7.670	-134.850	7.680	-139.100	7.690	-137.250	7.700	-129.600

7.710	-117.100	7.720	-100.820	7.730	-80.770	7.740	-57.460	7.750	-32.620
7.760	-8.010	7.770	15.471	7.780	37.660	7.790	58.690	7.800	78.450
7.810	96.280	7.820	111.410	7.830	123.450	7.840	132.300	7.850	137.600
7.860	139.200	7.870	136.400	7.880	129.500	7.890	119.250	7.900	106.520
7.910	92.010	7.920	75.840	7.930	57.720	7.940	37.920	7.950	17.550
7.960	-2.520	7.970	-21.820	7.980	-40.510	7.990	-59.170	8.000	-77.270
8.010	-92.750	8.020	-103.800	8.030	-110.140	8.040	-112.400	8.050	-110.570
8.060	-104.900	8.070	-96.650	8.080	-87.100	8.090	-75.870	8.100	-62.600
8.110	-48.260	8.120	-33.402	8.130	-17.496	8.140	-.406	8.150	16.910
8.160	32.890	8.170	46.180	8.180	56.360	8.190	64.200	8.200	71.036
8.210	77.420	8.220	83.650	8.230	90.380	8.240	97.560	8.250	103.810
8.260	107.540	8.270	108.040	8.280	105.310	8.290	99.760	8.300	92.090
8.310	82.870	8.320	72.570	8.330	61.660	8.340	50.380	8.350	38.570
8.360	25.950	8.370	12.300	8.380	-2.220	8.390	-16.790	8.400	-30.740
8.410	-44.040	8.420	-56.790	8.430	-68.800	8.440	-79.670	8.450	-89.080
8.460	-96.990	8.470	-103.440	8.480	-108.550	8.490	-112.400	8.500	-114.750
8.510	-114.970	8.520	-112.500	8.530	-107.260	8.540	-99.150	8.550	-88.090
8.560	-74.040	8.570	-57.180	8.580	-37.870	8.590	-16.850	8.600	5.270
8.610	28.380	8.620	51.910	8.630	74.260	8.640	93.720	8.650	109.450
8.660	121.100	8.670	128.200	8.680	130.400	8.690	127.700	8.700	120.100
8.710	107.250	8.720	89.000	8.730	66.080	8.740	39.720	8.750	11.180
8.760	-17.840	8.770	-45.240	8.780	-69.470	8.790	-90.020	8.800	-107.190
8.810	-121.150	8.820	-131.800	8.830	-139.700	8.840	-144.100	8.850	-143.450
8.860	-136.300	8.870	-123.250	8.880	-105.500	8.890	-84.000	8.900	-59.430
8.910	-32.320	8.920	-3.390	8.930	26.080	8.940	54.630	8.950	81.450
8.960	105.970	8.970	127.360	8.980	144.900	8.990	158.450	9.000	167.700
9.010	173.050	9.020	174.100	9.030	170.450	9.040	161.400	9.050	147.100
9.060	128.100	9.070	105.600	9.080	80.610	9.090	53.780	9.100	25.690
9.110	-2.990	9.120	-30.840	9.130	-55.700	9.140	-76.180	9.150	-92.250
9.160	-104.600	9.170	-113.500	9.180	-118.700	9.190	-119.850	9.200	-116.600
9.210	-109.750	9.220	-99.700	9.230	-86.700	9.240	-70.720	9.250	-52.310
9.260	-32.300	9.270	-11.850	9.280	7.790	9.290	26.030	9.300	42.200
9.310	54.980	9.320	63.940	9.330	70.120	9.340	74.920	9.350	79.170
9.360	82.530	9.370	83.690	9.380	81.760	9.390	76.760	9.400	69.500
9.410	60.990	9.420	51.460	9.430	40.670	9.440	28.463	9.450	15.039
9.460	1.330	9.470	-11.480	9.480	-22.580	9.490	-31.490	9.500	-38.230
9.510	-43.220	9.520	-46.510	9.530	-47.720	9.540	-46.320	9.550	-42.020
9.560	-35.140	9.570	-26.160	9.580	-15.980	9.590	-6.030	9.600	2.830
9.610	10.790	9.620	18.210	9.630	25.000	9.640	30.620	9.650	34.830
9.660	37.550	9.670	38.360	9.680	37.530	9.690	36.150	9.700	35.000
9.710	34.080	9.720	32.680	9.730	29.760	9.740	24.720	9.750	17.990
9.760	10.240	9.770	1.470	9.780	-8.120	9.790	-17.364	9.800	-25.280
9.810	-31.990	9.820	-38.050	9.830	-43.630	9.840	-48.640	9.850	-52.920
9.860	-56.130	9.870	-58.110	9.880	-58.775	9.890	-57.850	9.900	-55.430
9.910	-52.211	9.920	-49.047	9.930	-46.472	9.940	-44.559	9.950	-43.041
9.960	-41.607	9.970	-40.300	9.980	-39.340	9.990	-38.671	10.000	-38.390
10.010	-38.660	10.020	-39.390	10.030	-40.220	10.040	-39.990	10.050	-37.020
10.060	-30.160	10.070	-19.540	10.080	-6.290	10.090	8.012	10.100	22.020
10.110	34.520	10.120	44.600	10.130	52.290	10.140	57.070	10.150	57.350
10.160	52.400	10.170	43.200	10.180	30.860	10.190	14.480	10.200	-6.460
10.210	-30.710	10.220	-55.900	10.230	-79.890	10.240	-101.080	10.250	-118.280
10.260	-131.000	10.270	-139.800	10.280	-144.530	10.290	-144.400	10.300	-138.900
10.310	-128.900	10.320	-115.600	10.330	-99.380	10.340	-80.500	10.350	-59.300
10.360	-36.700	10.370	-14.033	10.380	7.322	10.390	26.120	10.400	41.880
10.410	54.940	10.420	65.430	10.430	73.060	10.440	77.580	10.450	79.020
10.460	78.150	10.470	76.330	10.480	74.550	10.490	72.620	10.500	70.240
10.510	68.080	10.520	66.548	10.530	64.570	10.540	60.990	10.550	55.970
10.560	49.720	10.570	41.880	10.580	32.073	10.590	20.610	10.600	8.510
10.610	-3.090	10.620	-12.810	10.630	-19.320	10.640	-22.380	10.650	-22.990
10.660	-22.430	10.670	-21.220	10.680	-19.420	10.690	-17.070	10.700	-13.870

10.710	-9.230	10.720	-2.640	10.730	6.040	10.740	16.190	10.750	26.350
10.760	35.050	10.770	41.830	10.780	46.702	10.790	49.470	10.800	50.360
10.810	50.340	10.820	50.408	10.830	50.610	10.840	50.520	10.850	49.790
10.860	47.700	10.870	43.270	10.880	35.840	10.890	25.300	10.900	12.090
10.910	-2.800	10.920	-18.566	10.930	-34.600	10.940	-50.370	10.950	-65.840
10.960	-80.335	10.970	-91.960	10.980	-99.770	10.990	-104.600	11.000	-107.640
11.010	-109.170	11.020	-108.390	11.030	-104.140	11.040	-95.750	11.050	-83.370
11.060	-67.750	11.070	-49.800	11.080	-30.170	11.090	-9.260	11.100	12.120
11.110	32.260	11.120	49.860	11.130	64.890	11.140	77.510	11.150	87.160
11.160	93.000	11.170	94.820	11.180	92.830	11.190	87.940	11.200	81.100
11.210	72.700	11.220	62.990	11.230	52.770	11.240	42.840	11.250	33.290
11.260	24.209	11.270	16.200	11.280	9.850	11.290	5.360	11.300	2.550
11.310	1.110	11.320	.640	11.330	.620	11.340	.990	11.350	2.150
11.360	4.400	11.370	7.500	11.380	10.880	11.390	14.340	11.400	17.730
11.410	20.259	11.420	21.710	11.430	23.080	11.440	24.980	11.450	26.965
11.460	28.160	11.470	27.938	11.480	26.599	11.490	25.182	11.500	24.690
11.510	25.360	11.520	26.670	11.530	28.040	11.540	28.850	11.550	28.360
11.560	26.130	11.570	22.440	11.580	17.770	11.590	12.500	11.600	7.220
11.610	2.620	11.620	-.720	11.630	-2.350	11.640	-2.390	11.650	-1.610
11.660	-.617	11.670	.920	11.680	3.420	11.690	6.430	11.700	9.400
11.710	12.390	11.720	15.210	11.730	16.890	11.740	16.320	11.750	13.120
11.760	7.362	11.770	-.919	11.780	-10.882	11.790	-20.966	11.800	-29.822
11.810	-36.880	11.820	-42.439	11.830	-47.303	11.840	-52.194	11.850	-57.220
11.860	-62.050	11.870	-66.320	11.880	-69.500	11.890	-70.980	11.900	-70.340
11.910	-67.710	11.920	-63.690	11.930	-58.970	11.940	-54.260	11.950	-50.090
11.960	-46.440	11.970	-42.270	11.980	-36.400	11.990	-28.420	12.000	-17.840
12.010	-3.320	12.020	15.710	12.030	37.772	12.040	60.600	12.050	82.640
12.060	102.520	12.070	118.400	12.080	129.100	12.090	134.500	12.100	134.700
12.110	129.250	12.120	117.700	12.130	100.550	12.140	78.700	12.150	52.740
12.160	23.730	12.170	-7.277	12.180	-38.660	12.190	-68.190	12.200	-93.800
12.210	-114.550	12.220	-130.000	12.230	-139.700	12.240	-143.600	12.250	-141.700
12.260	-134.700	12.270	-123.630	12.280	-109.630	12.290	-93.380	12.300	-75.610
12.310	-57.470	12.320	-39.980	12.330	-23.450	12.340	-7.610	12.350	7.842
12.360	23.170	12.370	38.750	12.380	54.421	12.390	69.210	12.400	82.510
12.410	94.600	12.420	105.830	12.430	115.870	12.440	124.100	12.450	129.750
12.460	132.200	12.470	131.100	12.480	125.800	12.490	114.650	12.500	96.600
12.510	72.050	12.520	42.340	12.530	9.520	12.540	-24.570	12.550	-57.940
12.560	-88.900	12.570	-116.450	12.580	-139.620	12.590	-156.300	12.600	-165.400
12.610	-167.950	12.620	-165.200	12.630	-156.800	12.640	-142.900	12.650	-124.950
12.660	-104.590	12.670	-82.260	12.680	-58.530	12.690	-34.850	12.700	-12.586
12.710	7.563	12.720	25.470	12.730	41.240	12.740	55.000	12.750	66.710
12.760	75.900	12.770	82.240	12.780	85.580	12.790	86.060	12.800	84.360
12.810	81.370	12.820	77.950	12.830	74.920	12.840	72.070	12.850	68.120
12.860	61.730	12.870	52.410	12.880	40.580	12.890	27.110	12.900	12.990
12.910	-.938	12.920	-14.149	12.930	-26.120	12.940	-36.660	12.950	-46.100
12.960	-54.490	12.970	-61.040	12.980	-64.720	12.990	-65.070	13.000	-62.030
13.010	-55.390	13.020	-45.610	13.030	-34.020	13.040	-21.900	13.050	-10.038
13.060	1.220	13.070	12.043	13.080	22.360	13.090	31.390	13.100	38.670
13.110	44.500	13.120	49.380	13.130	53.370	13.140	56.070	13.150	57.010
13.160	56.100	13.170	53.440	13.180	49.390	13.190	44.480	13.200	38.620
13.210	31.340	13.220	22.296	13.230	11.442	13.240	-.454	13.250	-11.920
13.260	-21.780	13.270	-29.890	13.280	-36.501	13.290	-41.440	13.300	-44.720
13.310	-47.160	13.320	-49.350	13.330	-50.840	13.340	-50.970	13.350	-49.600
13.360	-46.800	13.370	-42.500	13.380	-36.830	13.390	-30.440	13.400	-23.784
13.410	-16.857	13.420	-9.740	13.430	-2.940	13.440	3.030	13.450	8.275
13.460	13.000	13.470	16.894	13.480	19.980	13.490	23.010	13.500	26.500
13.510	30.180	13.520	33.240	13.530	34.870	13.540	34.550	13.550	32.320
13.560	28.560	13.570	23.711	13.580	18.285	13.590	12.739	13.600	7.634
13.610	3.535	13.620	.530	13.630	-1.720	13.640	-3.700	13.650	-5.810
13.660	-7.823	13.670	-8.870	13.680	-8.390	13.690	-6.510	13.700	-3.800

13.710	-.966	13.720	1.590	13.730	3.890	13.740	5.838	13.750	7.000
13.760	6.950	13.770	5.750	13.780	3.714	13.790	1.142	13.800	-1.768
13.810	-4.710	13.820	-7.390	13.830	-9.770	13.840	-11.591	13.850	-12.200
13.860	-11.340	13.870	-9.560	13.880	-7.475	13.890	-5.222	13.900	-2.745
13.910	-.145	13.920	2.271	13.930	4.110	13.940	5.130	13.950	5.458
13.960	5.302	13.970	4.490	13.980	2.820	13.990	.550	14.000	-2.179
14.010	-5.599	14.020	-9.860	14.030	-14.591	14.040	-19.160	14.050	-23.000
14.060	-25.570	14.070	-26.500	14.080	-25.680	14.090	-23.220	14.100	-19.470
14.110	-14.900	14.120	-9.890	14.130	-4.550	14.140	1.170	14.150	7.146
14.160	13.180	14.170	19.140	14.180	24.620	14.190	29.140	14.200	32.248
14.210	33.650	14.220	33.560	14.230	32.700	14.240	31.680	14.250	30.620
14.260	29.190	14.270	26.900	14.280	23.390	14.290	18.700	14.300	12.990
14.310	6.485	14.320	-.483	14.330	-7.492	14.340	-13.966	14.350	-19.350
14.360	-23.480	14.370	-26.700	14.380	-29.506	14.390	-32.120	14.400	-34.510
14.410	-36.650	14.420	-38.370	14.430	-39.190	14.440	-38.730	14.450	-36.870
14.460	-33.660	14.470	-29.350	14.480	-24.120	14.490	-18.070	14.500	-11.270
14.510	-3.950	14.520	3.510	14.530	10.750	14.540	17.290	14.550	22.640
14.560	26.370	14.570	28.290	14.580	28.400	14.590	26.850	14.600	23.910
14.610	19.740	14.620	14.520	14.630	8.550	14.640	1.920	14.650	-5.490
14.660	-13.744	14.670	-22.525	14.680	-31.200	14.690	-39.120	14.700	-45.690
14.710	-50.430	14.720	-53.190	14.730	-54.160	14.740	-53.630	14.750	-51.870
14.760	-49.020	14.770	-45.030	14.780	-39.870	14.790	-33.640	14.800	-26.470
14.810	-18.410	14.820	-9.659	14.830	-.613	14.840	8.164	14.850	16.090
14.860	22.800	14.870	28.240	14.880	32.530	14.890	35.820	14.900	38.130
14.910	39.330	14.920	39.300	14.930	37.950	14.940	35.310	14.950	31.400
14.960	26.350	14.970	20.500	14.980	14.210	14.990	7.653		

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APPENDIX C

POTENTIAL DIFFERENCES BETWEEN DYNAMIC AND STATIC MASS DISTRIBUTIONS FOR THE SAME COMPUTER MODEL

The analysis of nuclear power plant structures, systems and components often requires that static analysis solutions and dynamic analysis solutions for the same computer model be combined to produce the total stress state, for comparison to a design allowable stress. One such combination is deadweight plus seismic load. To produce accurate load combination results, it is important that the mass distribution for the static analysis and the dynamic analysis be comparable and sufficiently refined. During the course of the present study (see Section 2.3.2), convergence of the Lindley-Yow plus missing mass in-phase response component to the static analysis solution for total mass times ZPA was tested for the BM3 piping model used in the numerical studies. The first attempt to correlate these two solutions was unsuccessful, because the piping computer code, based on SAP V, treats the distributed mass of the piping model differently in static analyses and dynamic analyses. The dynamic analysis treats the distributed piping mass as lumped masses at the node points, based on adjacent pipe element lengths. The static analysis treats the distributed piping mass times acceleration as a distributed load along the length of the pipe element. The static analysis procedure is the more accurate representation for calculation of stresses and reactions at supports. However, for the dynamic analysis solution, a finite number of mass degrees of freedom are defined by assigning the mass to the locations of the node points.

In dynamic analysis, the definition of node points is primarily driven by the physical characteristics of the piping system (support points, branch points, and in-line components) and by the dynamic behavior which is to be predicted (frequency range and mode shapes). A third criterion for definition of node points should also be included: sufficient refinement for accurate prediction of stresses and support reactions.

Tables C-1 and C-2 present a comparison between two static analysis solutions, for BM3 support reactions and pipe end moments, respectively. The column labeled "static/dyn" are the results for the dynamic lumped mass representation multiplied by ZPA. The column labeled "static/sta" are the results for the static distributed mass representation multiplied by ZPA. Both analyses utilized the same computer model and computer code.

Examination of Table C-1 indicates that reaction forces F_x, F_y, F_z are in reasonable agreement; however, moment reactions at the fixed support points (nodes 1, 31, 38) have significant differences (>10%). Table C-2 indicates that the pipe end moments for elements in the vicinity of the fixed support points also have significant differences (>10%). Two effects contribute to poor correlation of the moment. In the dynamic analysis, mass apportioned to the fixed supports (nodes 1, 31, 38) do not produce any moment effect. Also, concentrated loads (i.e., lumped masses) do not generally produce the same moments as a distributed load. Both of these effects can be minimized in the dynamic analysis by defining of a sufficient number of node points in the vicinity of fixed supports. During development of the mathematical model, a sensitivity study should be conducted to ensure that the node point distribution is sufficient to produce an accurate static solution. Successive refinement of the node point distribution should be performed until there is reasonable correlation between results generated with the dynamic lumped mass representation and the static distributed mass representation.

Some existing computer codes for piping analysis may be less susceptible to this condition because corrections have been built into the code. It is important for the analyst to understand the treatment of mass and ensure that an adequate lumped mass distribution has been defined for the dynamic analysis.

Support Reactions of Static Analyses due to Different Mass Distributions

Forces in lbs
Moments in in-lbs

1 % Damping

NODE NO.	REAC. TYPE	ZPA static/dyn	ZPA static/sta
1	Fx	-46.21	-50.50
1	Fy	0.66	0.66
1	Fz	0.26	0.01
1	Mx	10.53	32.80
1	My	-329.25	-478.00
1	Mz	158.43	-823.00
4	Fx	-102.75	-98.00
4	Fz	-5.49	-4.26
7	Fy	-0.01	-0.85
11	Fy	-1.17	-0.12
11	Fz	37.65	36.00
15	Fx	-474.26	-475.00
17	Fy	3.78	3.84
17	Fz	-35.17	-34.40
36	Fy	8.92	8.70
36	Fz	31.68	34.80
38	Fx	-767.29	-770.00
38	Fy	-11.24	-11.20
38	Fz	-31.17	-35.60
38	Mx	-189.06	-195.00
38	My	-2202.83	-2520.00
38	Mz	804.58	799.00
23	Fx	-304.11	-297.00
23	Fy	-0.68	-0.15
31	Fx	-56.51	-60.60
31	Fy	-0.25	-0.92
31	Fz	2.24	3.49
31	Mx	219.27	344.00
31	My	580.84	502.00
31	Mz	1702.48	2200.00

Table C-1

Pipe End Moments of Static Analyses due to Different Mass Distribution

Resultant Moments in in-lbs

1 % Damping

ELEM. NO.	ZPA static/dyn	ZPA static/sta
1	357.75	961.37
2	450.75	808.88
3	995.58	1241.14
4	327.00	526.13
5	241.67	303.25
6	868.02	627.91
7	725.62	486.36
8	354.59	179.07
9	502.09	498.37
10	912.98	836.15
11	1617.18	1642.18
12	3951.97	3927.76
13	2956.60	2925.35
14	8613.31	8664.29
15	475.77	503.56
16	467.76	445.32
17	310.22	302.07
18	706.10	755.46
19	471.56	519.70
20	2028.91	2009.32
21	580.58	524.32
22	1971.49	1987.20
23	1167.59	1191.47
24	1123.54	1119.43
25	922.09	788.22
26	657.25	536.09
27	340.76	257.71
28	600.98	502.43
29	710.13	578.43
30	1812.15	2279.14
31	7523.23	7288.41
32	9107.41	8828.37
33	10884.84	10502.09
34	2199.42	2703.06
35	4815.86	5424.33
36	1247.94	1402.92
37	2352.78	2649.47

Table C-2

APPENDIX D

TABULATION OF MODE CORRELATION COEFFICIENTS FOR THE DOUBLE SUM COMBINATION (DSC) AND THE COMPLETE QUADRATIC COMBINATION (CQC) METHODS

1% DAMPING

MODE CORRELATION COEFFICIENTS

f_1	f_1/f_2	DSC (T = 10 sec)	DSC (T = 20 sec)	DSC (T = 1000 sec) and CQC
2 Hz	1.0	1.0	1.0	1.0
	.95	.54	.36	.15
	.91	.22	.12	.04
	.87	.11	.06	.02
5 Hz	1.0	1.0	1.0	1.0
	.95	.32	.24	.15
	.91	.10	.07	.04
	.87	.05	.03	.02
10 Hz	1.0	1.0	1.0	1.0
	.95	.23	.19	.15
	.91	.07	.06	.04
	.87	.03	.03	.02
20 Hz	1.0	1.0	1.0	1.0
	.95	.19	.17	.15
	.91	.06	.05	.04
	.87	.03	.02	.02
30 Hz	1.0	1.0	1.0	1.0
	.95	.18	.16	.15
	.91	.05	.05	.04
	.87	.02	.02	.02

2% DAMPING

MODE CORRELATION COEFFICIENTS

f_1	f_1/f_2	DSC (T = 10 sec)	DSC (T = 20 sec)	DSC (T = 1000 sec) and CQC
2 Hz	1.0	1.0	1.0	1.0
	.95	.69	.58	.41
	.91	.35	.25	.15
	.87	.19	.13	.07
	.83	.12	.08	.04
5 Hz	1.0	1.0	1.0	1.0
	.95	.55	.49	.41
	.91	.23	.19	.15
	.87	.12	.09	.07
	.83	.07	.06	.04
10 Hz	1.0	1.0	1.0	1.0
	.95	.49	.45	.41
	.91	.19	.17	.15
	.87	.09	.08	.07
	.83	.06	.05	.04
20 Hz	1.0	1.0	1.0	1.0
	.95	.45	.43	.41
	.91	.17	.16	.15
	.87	.08	.08	.07
	.83	.05	.05	.04
30 Hz	1.0	1.0	1.0	1.0
	.95	.44	.43	.41
	.91	.16	.16	.15
	.87	.08	.08	.07
	.83	.05	.05	.04

5% DAMPING

MODE CORRELATION COEFFICIENTS

f_1	f_1/f_2	DSC (T = 10 sec)	DSC (T = 20 sec)	DSC (T = 1000 sec) and CQC
2 Hz	1.0	1.0	1.0	1.0
	.95	.88	.86	.82
	.91	.65	.60	.53
	.87	.45	.39	.33
	.83	.32	.27	.22
	.79	.23	.19	.15
	.75	.17	.14	.11
5 Hz	1.0	1.0	1.0	1.0
	.95	.85	.83	.82
	.91	.58	.56	.53
	.87	.38	.36	.33
	.83	.26	.24	.22
	.79	.18	.17	.15
	.75	.13	.12	.11
10 Hz	1.0	1.0	1.0	1.0
	.95	.83	.83	.82
	.91	.56	.54	.53
	.87	.36	.34	.33
	.83	.24	.23	.22
	.79	.17	.16	.15
	.75	.12	.12	.11

5% DAMPING

MODE CORRELATION COEFFICIENTS (CONT'D)

f_1	f_1/f_2	DSC (T = 10 sec)	DSC (T = 20 sec)	DSC (T = 1000 sec) and CQC
20 Hz	1.0	1.0	1.0	1.0
	.95	.83	.82	.82
	.91	.54	.53	.53
	.87	.34	.34	.33
	.83	.23	.22	.22
	.79	.16	.16	.15
	.75	.12	.11	.11
30 Hz	1.0	1.0	1.0	1.0
	.95	.82	.82	.82
	.91	.54	.53	.53
	.87	.34	.34	.33
	.83	.23	.22	.22
	.79	.16	.15	.15
	.75	.12	.11	.11

10% DAMPING

MODE CORRELATION COEFFICIENTS

f_1	f_1/f_2	DSC (T = 10 sec)	DSC (T = 20 sec)	DSC (T = 1000 sec) and CQC
2 Hz	1.0	1.0	1.0	1.0
	.95	.96	.95	.95
	.91	.86	.84	.81
	.87	.72	.70	.66
	.83	.60	.56	.52
	.79	.48	.45	.41
	.75	.39	.36	.32
	.72	.32	.30	.26
	.68	.27	.24	.21
	.65	.22	.20	.17
	.62	.19	.17	.14
	.59	.16	.15	.12
	.57	.14	.13	.10
	.54	.12	.11	.09
	.51	.11	.10	.08

10% DAMPING

MODE CORRELATION COEFFICIENTS (CONT'D)

f_1	f_1/f_2	DSC (T = 10 sec)	DSC (T = 20 sec)	DSC (T = 1000 sec) and CQC
5 Hz	1.0	1.0	1.0	1.0
	.95	.95	.95	.95
	.91	.83	.83	.81
	.87	.69	.68	.66
	.83	.56	.54	.52
	.79	.45	.43	.41
	.75	.36	.35	.32
	.72	.29	.28	.26
	.68	.24	.23	.21
	.65	.20	.19	.17
	.62	.17	.16	.14
	.59	.15	.14	.12
	.57	.13	.12	.10
	.54	.11	.11	.09
	.51	.10	.09	.08

10% DAMPING

MODE CORRELATION COEFFICIENTS (CONT'D)

f_1	f_1/f_2	DSC (T = 10 sec)	DSC (T = 20 sec)	DSC (T = 1000 sec) and CQC
10 Hz	1.0	1.0	1.0	1.0
	.95	.95	.95	.95
	.91	.83	.82	.81
	.87	.68	.67	.66
	.83	.54	.54	.52
	.79	.43	.43	.41
	.75	.35	.34	.32
	.72	.28	.28	.26
	.68	.23	.23	.21
	.65	.19	.19	.17
	.62	.16	.16	.14
	.59	.14	.14	.12
	.57	.12	.12	.10
	.54	.11	.10	.09
	.51	.09	.09	.08

10% DAMPING

MODE CORRELATION COEFFICIENTS (CONT'D)

f_1	f_1/f_2	DSC (T = 10 sec)	DSC (T = 20 sec)	DSC (T = 1000 sec) and CQC
20 Hz	1.0	1.0	1.0	1.0
	.95	.95	.95	.95
	.91	.82	.82	.81
	.87	.67	.67	.66
	.83	.54	.53	.52
	.79	.43	.42	.41
	.75	.34	.34	.32
	.72	.28	.27	.26
	.68	.23	.22	.21
	.65	.19	.19	.17
	.62	.16	.16	.14
	.59	.14	.14	.12
	.57	.12	.12	.10
	.54	.11	.10	.09
	.51	.09	.09	.08

10% DAMPING

MODE CORRELATION COEFFICIENTS (CONT'D)

f_1	f_1/f_2	DSC (T = 10 sec)	DSC (T = 20 sec)	DSC (T = 1000 sec) and CQC
30 Hz	1.0	1.0	1.0	1.0
	.95	.95	.95	.95
	.91	.82	.82	.81
	.87	.67	.67	.66
	.83	.53	.53	.52
	.79	.42	.42	.41
	.75	.34	.34	.32
	.72	.28	.27	.26
	.68	.23	.22	.21
	.65	.19	.19	.17
	.62	.16	.16	.14
	.59	.14	.14	.12
	.57	.12	.12	.10
	.54	.10	.10	.09
	.51	.09	.09	.08

APPENDIX E

COMPARISON OF TIME HISTORY SOLUTION METHODS: MODE SUPERPOSITION VS. DIRECT INTEGRATION

Comparison of Mode Superposition vs. Direct Integration Time History

In mode superposition time history analysis, the residual or "missing" mass effect must be calculated and algebraically summed with the modal responses. To utilize modal time history analysis efficiently, only modal responses with frequencies up to the ZPA frequency need to be calculated; the "missing mass" not participating in these modes should be treated as an additional mode, with its peak response calculated from a static analysis of the system for ZPA times "missing mass" (as defined in Appendix I of report). The time variation for the missing mass mode is obtained by scaling this peak response to the input time history. Clearly, knowledge of the response spectrum is very useful when mode superposition time history is being performed.

The value of the response spectrum is perhaps even more significant when a direct integration time history analysis is being conducted. The spectrum provides useful information for (1) the definition of the integration time step and (2) the development of α - β damping coefficients to simulate constant modal damping. The integration time step must be sufficiently small to capture responses which oscillate at all frequencies below the ZPA frequency. In addition, knowing the frequency range of spectral amplification can help in the selection of target frequencies to obtain a best fit α - β combination for approximating constant modal damping.

For the direct integration time history analyses of BM3, α and β were determined by specifying the target modal damping at the frequency of the fundamental mode (2.9 Hz) of the BM3 model and at an intermediate frequency between this frequency and f_{ZPA} (16.5 Hz). A frequency of 11.5 Hz was selected, in order to achieve the best fit over the 2.9 Hz to 16.5 Hz range of interest. This resulted in a minimum damping equal to 80% of the target modal damping, which occurs at ~ 5.7 Hz, and a maximum damping equal to 130% of the target modal damping, which occurs at 16.5 Hz. Sensitivity to variations in the damping value diminishes as f_{ZPA} is approached; consequently, somewhat higher effective modal damping in the high frequency end of the amplified range of the spectrum has relatively negligible effect on the total dynamic solution.

In the case of the BM3 model and loading, comparison between the direct integration solution and the mode superposition plus missing mass solution indicates close agreement for both 1% and 5% damping. See Tables E-1 through E-4. The missing mass contribution to the mode superposition solution is significant. Without accounting for this contribution, one might conclude that direct integration is somewhat inaccurate when compared to mode superposition because of its inherent inability to represent constant modal damping. Table E-5 shows the missing mass, representing the residual after 31 modes (> 70 Hz). By including the missing mass contribution, excellent correlation was obtained between the complete mode superposition solution and the direct integration solution.

Based on this comparative study, it is plausible that criticism of direct integration methods may be rooted in three potential sources for differences between direct integration and mode superposition solutions:

1) integration time step too large in the direct integration analysis;
2) poor definition of α - β coefficients in the direct integration analysis;
3) failure to include the "missing mass" contribution in the mode superposition analysis.

Table E-1
Comparison of Peak Support Reaction Forces of BM3
Due to Different Approaches

Forces in lbs
Moments in in-lbs **1 % Damping**

NODE NO.	REAC. TYPE	MODAL (31 modes)	TIME at sec.	MODAL (31m) Plus M.M.	TIME at sec	DIRECT INTEGRATION	TIME at sec.
1	Fx	8.40	7.34	43.71	7.36	43.63	7.35
1	Fy	10.14	7.34	4.36	4.56	3.35	4.69
1	Fz	1.60	7.48	1.60	7.48	1.59	7.35
1	Mx	49.98	6.68	49.88	6.68	45.14	6.68
1	My	776.73	7.36	776.40	7.36	778.86	7.35
1	Mz	193.46	4.54	278.42	7.36	279.55	7.35
4	Fx	68.75	7.34	116.79	7.36	117.78	7.35
4	Fz	19.55	7.34	20.01	7.34	21.03	7.35
7	Fy	13.31	12.90	13.27	12.90	13.78	12.89
11	Fy	13.31	12.72	13.31	12.72	13.74	12.73
11	Fz	81.14	7.36	81.34	7.36	82.49	7.35
15	Fx	713.32	7.36	731.47	7.36	740.29	7.35
17	Fy	25.61	11.00	25.60	11.00	27.45	11.01
17	Fz	64.26	7.34	65.36	7.34	66.32	7.35
36	Fy	46.31	4.80	46.69	4.80	47.26	4.80
36	Fz	48.85	7.18	42.12	3.88	41.80	4.46
38	Fx	128.79	4.48	732.18	7.36	736.41	7.35
38	Fy	43.52	7.36	43.44	7.36	42.51	7.36
38	Fz	31.40	4.48	29.95	4.48	29.49	4.48
38	Mx	720.04	7.34	719.05	7.34	719.65	7.33
38	My	2142.00	4.48	2084.97	4.48	2055.80	4.48
38	Mz	3088.90	7.36	3085.87	7.36	3018.90	7.36
23	Fx	259.22	4.48	259.59	4.48	265.71	4.47
23	Fy	26.04	7.38	26.08	7.38	24.60	7.38
31	Fx	23.24	7.34	55.05	7.34	55.78	7.35
31	Fy	14.21	10.56	14.17	10.56	14.20	10.56
31	Fz	16.07	11.84	16.08	11.84	16.90	11.83
31	Mx	1136.90	11.84	1137.30	11.84	1173.80	11.84
31	My	608.43	7.34	612.38	7.34	620.26	7.35
31	Mz	1767.90	7.34	1773.20	7.34	1776.60	7.35

Table E-2
Comparison of Peak Pipe Resultant Moments of BM3
Due to Different Approaches

Moments in in-lbs — 1 % Damping

ELEM. No.	MODAL (31 modes)	TIME at sec.	MODAL (31m) Plus M.M.	TIME at sec	DIRECT INTEGRATION	TIME at sec.
1	823.40	7.36	790.95	7.36	791.02	7.35
2	818.58	7.36	836.77	7.36	838.40	7.35
3	1412.22	7.36	1429.28	7.36	1441.59	7.35
4	762.56	7.36	763.87	7.36	764.19	7.35
5	598.91	7.36	597.98	7.36	602.54	7.35
6	2134.36	7.36	2133.18	7.36	2141.09	7.35
7	1876.38	7.36	1877.81	7.36	1880.06	7.35
8	852.96	7.38	852.74	7.38	832.43	4.65
9	1415.46	7.38	1415.21	7.38	1438.67	4.66
10	1945.32	7.36	1946.24	7.36	1967.00	7.35
11	3826.15	7.36	3829.46	7.36	3804.39	7.35
12	8440.47	7.36	8428.48	7.36	8437.39	7.35
13	6741.68	7.36	6741.03	7.36	6738.07	7.35
14	13241.93	7.36	13219.91	7.36	13373.34	7.35
15	2290.84	7.36	2293.75	7.36	2364.57	4.67
16	1698.69	10.56	1696.61	10.56	1697.84	10.56
17	2455.47	10.66	2455.62	10.66	2492.35	10.66
18	2360.13	7.36	2363.76	7.36	2349.31	7.35
19	1623.90	7.36	1625.77	7.36	1623.68	7.35
20	7539.21	7.36	7544.81	7.36	7484.45	7.35
21	1595.61	7.36	1590.16	7.36	1564.96	7.35
22	2042.84	10.56	2046.41	10.56	2036.58	10.56
23	1800.75	10.56	1807.75	10.56	1802.73	10.56
24	1522.11	10.56	1518.31	10.56	1514.11	10.56
25	926.28	7.34	922.51	7.34	915.35	7.34
26	665.57	7.34	677.35	7.34	674.46	7.34
27	399.00	7.34	390.71	7.34	387.38	7.34
28	742.39	7.34	737.42	7.34	734.60	7.34
29	824.73	7.34	830.76	7.34	825.51	7.34
30	1988.96	7.34	1994.56	7.34	2042.35	7.35
31	7437.70	10.56	7428.79	10.56	7377.43	10.56
32	8931.45	7.34	8785.32	7.34	8703.23	7.35
33	11421.16	7.34	10970.06	7.34	10866.98	7.34
34	6096.62	7.36	5993.82	7.36	5966.82	7.35
35	6633.91	7.36	6788.22	7.36	6757.74	7.35
36	1942.86	7.36	1874.84	7.36	1892.65	7.35
37	3538.74	7.36	3501.06	7.36	3487.86	7.35

Table E-3
Comparison of Peak Support Reaction Forces of BM3
Due to Different Approaches

Forces in lbs
Moments in in-lbs **5 % Damping**

NODE NO.	REAC. TYPE	MODAL (31 modes)	TIME at sec.	MODAL (31m) Plus M.M.	TIME at sec	DIRECT INTEGRATION	TIME at sec.
1	Fx	7.87	7.36	44.22	7.36	44.19	7.35
1	Fy	11.33	7.34	3.86	7.26	2.24	7.39
1	Fz	0.97	7.50	0.97	7.50	1.02	7.49
1	Mx	19.11	9.04	18.89	9.04	18.84	4.51
1	My	695.11	7.36	694.80	7.36	703.87	7.36
1	Mz	156.85	7.38	240.22	7.36	239.67	7.37
4	Fx	65.32	7.36	113.70	7.36	113.82	7.35
4	Fz	13.38	7.36	13.85	7.36	14.07	7.36
7	Fy	8.01	7.60	7.95	7.60	8.26	7.61
11	Fy	7.13	12.74	7.13	12.74	7.54	12.73
11	Fz	71.69	7.36	71.89	7.36	72.60	7.36
15	Fx	672.79	7.36	690.97	7.36	692.19	7.36
17	Fy	20.15	7.38	20.14	7.38	20.26	7.38
17	Fz	58.91	7.36	60.05	7.36	60.27	7.36
36	Fy	37.58	7.38	38.70	7.38	39.84	7.37
36	Fz	50.25	12.60	42.33	12.60	41.88	12.61
38	Fx	136.14	7.34	740.68	7.36	744.53	7.35
38	Fy	38.25	7.38	38.18	7.38	39.69	7.37
38	Fz	31.43	7.18	29.85	7.18	29.61	7.19
38	Mx	466.39	7.36	465.42	7.36	481.87	7.49
38	My	2143.30	7.18	2080.83	7.18	2065.30	7.19
38	Mz	2715.60	7.38	2713.20	7.38	2820.50	7.37
23	Fx	270.09	12.60	270.50	12.60	268.35	12.60
23	Fy	10.54	7.38	10.58	7.38	10.97	6.57
31	Fx	22.57	7.34	54.46	7.36	55.10	7.35
31	Fy	9.62	7.38	9.76	7.38	10.16	7.50
31	Fz	8.66	7.28	8.66	7.28	9.02	7.28
31	Mx	559.28	12.60	560.28	12.60	598.88	7.29
31	My	578.29	7.34	582.28	7.34	586.45	7.35
31	Mz	1679.80	7.34	1685.20	7.34	1685.60	7.35

Table E-4
Comparison of Peak Pipe Resultant Moments of BM3
Due to Different Approaches

Moments in in-lbs | 5 % Damping

ELEM. No.	MODAL (31 modes)	TIME at sec.	MODAL (31m) Plus M.M.	TIME at sec	DIRECT INTEGRATION	TIME at sec.
1	745.97	7.36	713.05	7.36	721.44	7.36
2	746.03	7.36	765.02	7.36	772.40	7.36
3	1306.36	7.36	1323.65	7.36	1326.79	7.36
4	683.45	7.36	684.42	7.36	693.18	7.36
5	529.22	7.36	528.66	7.36	538.01	7.37
6	1894.42	7.36	1893.29	7.36	1919.87	7.37
7	1663.22	7.36	1664.64	7.36	1695.52	7.37
8	765.19	7.36	764.93	7.36	781.15	7.37
9	1320.17	7.36	1319.86	7.38	1362.63	7.37
10	1713.20	7.36	1714.12	7.36	1730.62	7.36
11	3504.53	7.36	3507.84	7.36	3557.59	7.37
12	7662.41	7.36	7650.43	7.36	7733.23	7.36
13	6075.86	7.36	6075.21	7.36	6149.10	7.36
14	12509.76	7.36	12487.74	7.36	12525.94	7.36
15	2027.62	7.38	2030.00	7.38	2061.56	7.37
16	1057.72	7.26	1057.35	7.26	1032.69	7.26
17	1496.87	7.38	1495.69	7.38	1523.29	7.37
18	2023.46	7.36	2027.05	7.36	2107.67	7.37
19	1368.94	7.38	1370.50	7.38	1433.54	7.37
20	6607.12	7.36	6612.74	7.36	6886.72	7.37
21	1340.38	7.38	1337.38	7.38	1397.33	7.37
22	1789.34	7.36	1803.08	7.36	1822.39	7.35
23	1339.89	7.36	1352.91	7.36	1368.99	7.36
24	1236.93	7.36	1229.48	7.36	1232.76	7.36
25	962.11	7.34	958.32	7.34	952.48	7.34
26	677.97	7.34	689.97	7.34	687.08	7.34
27	328.88	12.6	322.88	12.6	320.59	12.6
28	630.00	7.36	624.60	7.36	635.75	7.35
29	731.43	7.36	737.73	7.36	749.77	7.35
30	1777.89	7.34	1784.26	7.34	1798.77	7.35
31	7221.12	7.36	7194.98	7.36	7240.74	7.35
32	8935.01	7.34	8768.03	7.34	8747.80	7.35
33	11395.76	7.34	10912.71	7.34	10913.38	7.35
34	5486.76	7.36	5338.54	7.36	5519.24	7.37
35	6167.02	7.36	6348.71	7.36	6378.37	7.36
36	1835.19	7.36	1755.84	7.36	1763.67	7.36
37	3308.12	7.36	3263.83	7.36	3278.70	7.36

Table E-5
Nodal Masses Participating In 31 Mode Combination
(lbs-sec**2/in)
Note: Negative missing mass indicates 31 mode mass exceeds total mass.

Node No.	Total Mass	Participating Mass of 31 modes	Missing Mass after 31 modes
1	0.017	0.000	0.017
2	0.026	0.000	0.026
3	0.212	0.001	0.211
4	0.227	0.000	0.227
5	0.031	0.024	0.007
6	0.165	0.166	-0.001
7	0.165	0.168	-0.003
8	0.092	0.093	-0.001
9	0.267	0.270	-0.003
10	0.275	0.277	-0.002
11	0.182	0.183	-0.001
12	0.174	0.174	0.000
13	0.092	0.092	0.000
14	0.059	0.053	0.006
15	0.101	0.008	0.093
16	0.059	0.055	0.004
17	0.105	0.106	-0.001
18	0.268	0.272	-0.004
19	0.180	0.183	-0.003
20	0.073	0.073	0.000
21	0.207	0.170	0.037
22	0.088	0.066	0.022
23	0.043	0.038	0.005
24	0.052	0.053	-0.001
25	0.197	0.202	-0.005
26	0.178	0.189	-0.011
27	0.048	0.053	-0.005
28	0.063	0.072	-0.009
29	0.048	0.051	-0.003
30	0.175	0.165	0.010
31	0.158	0.000	0.158
32	0.163	0.150	0.013
33	0.276	0.280	-0.004
34	0.780	0.881	-0.101
35	0.687	0.090	0.597
36	0.687	0.008	0.679
37	1.170	0.007	1.163
38	0.585	0.000	0.585

APPENDIX F

USE OF RESPONSE SPECTRUM GENERATION SDOF OSCILLATOR RESPONSES TO ESTABLISH THE THRESHOLD FREQUENCY FOR IN-PHASE MODAL RESPONSE

Methodology for Identification of the Lowest-Frequency SDOF Oscillator which Responds In-phase with the Input Time History during Response Spectrum Generation

A key element in the utilization of the Response Spectrum Method to predict peak dynamic response due to a seismic time history input is establishing appropriate phase relationships between the individual peak modal responses, so that the result of modal combination provides a reasonable estimate of the peak dynamic response.

During the generation of a response spectrum from a ground or in-structure time history record, the complete time history of each SDOF oscillator response is calculated and processed to identify the peak response. This peak response becomes a single point on the response spectrum plot. Associated with each SDOF oscillator peak response is the time of occurrence and direction of the peak response. This information is typically not retained since it is not needed to generate the response spectrum. However, valuable conclusions can be derived from this information by comparison to the time and direction of the peak acceleration in the time history record.

The lowest SDOF oscillator frequency (f_{ip}) for which the time and direction of peak response coincides with the time and direction of the peak of the input time history represents the onset of in-phase response with the input, provided all higher-frequency SDOF oscillator responses exhibit the same behavior; i.e., for $f \geq f_{ip}$, all SDOF oscillator peak responses occur at the same time and in the same direction as the peak of the input time history. To further verify that in-phase response exists, a comparison of the crossings of the acceleration equal to zero datum between the input time history and SDOF oscillator time history response should be performed for SDOF oscillator frequencies in the vicinity of f_{ip}.

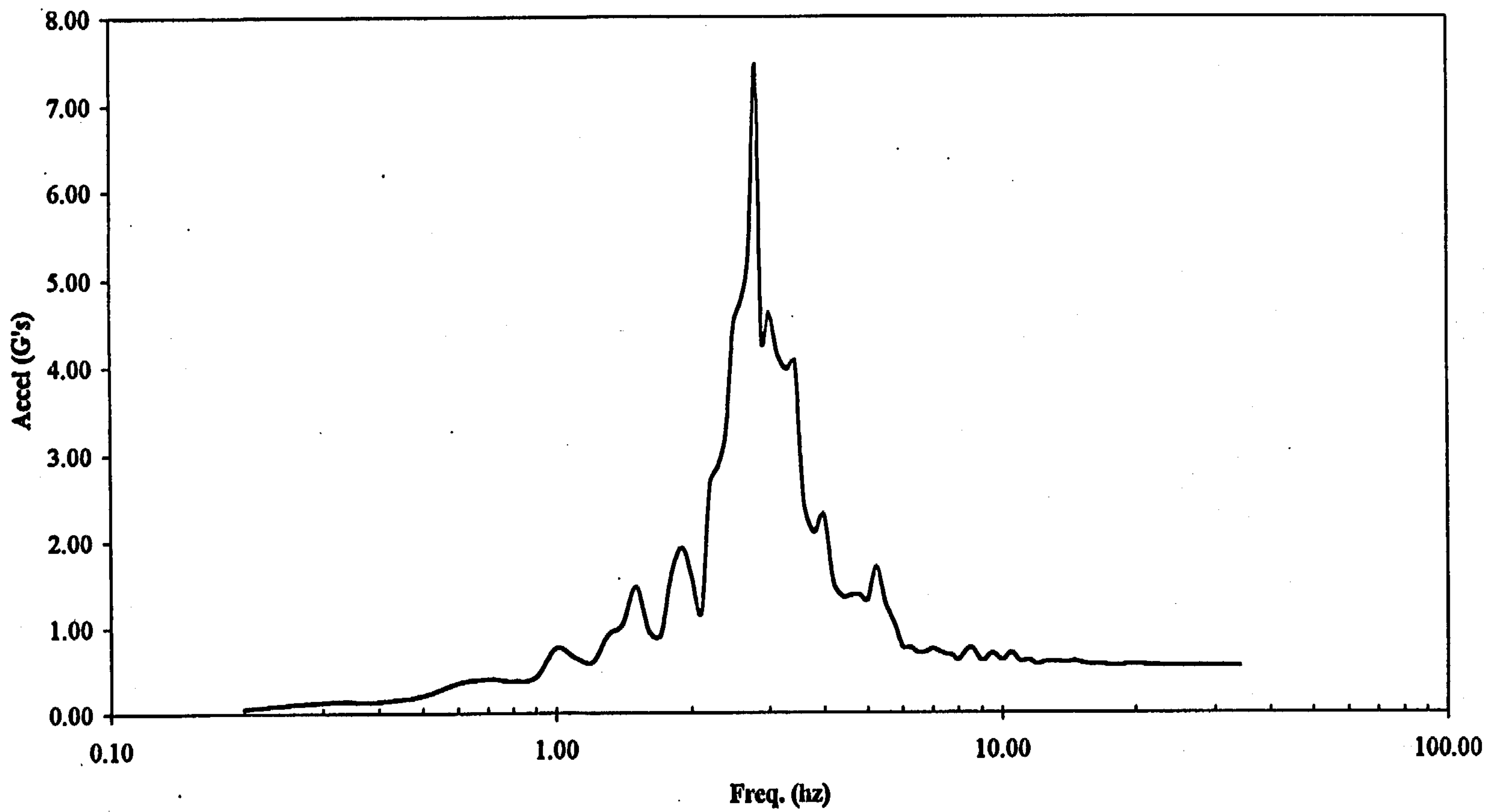
Once f_{ip} is established by this procedure, which can be fully automated and made a part of the response spectrum generation algorithm, it is straightforward to specify in the modal combination methodology that all structural modes with frequency $\geq f_{ip}$ are to be combined algebraically. Application of this approach to the BM3 time history record for 1% and 5% damping produced the following results:

	f_{ip}	f_{zpa}
1% damping	9.5 HZ	16.5 HZ
5% damping	7.5 HZ	16.5 HZ

Algebraic combination of structural modal responses for $f \geq f_{zpa}$ is accomplished by the "missing mass" procedure using the ZPA. In the frequency range f_{ip} to f_{zpa}, individual structural modal responses are algebraically combined with the "missing mass" contribution. Consequently, the response of all modes with $f \geq f_{ip}$ are combined algebraically.

Numerical Results

Numerical data for the 1% and 5% damping cases are provided: the response spectrum plots and tables showing the peak spectral acceleration and associated time of occurrence for the frequency range 0.2 to 34 Hz. In-phase response was verified by comparing the crossings of the acceleration equal to zero datum, as discussed above.

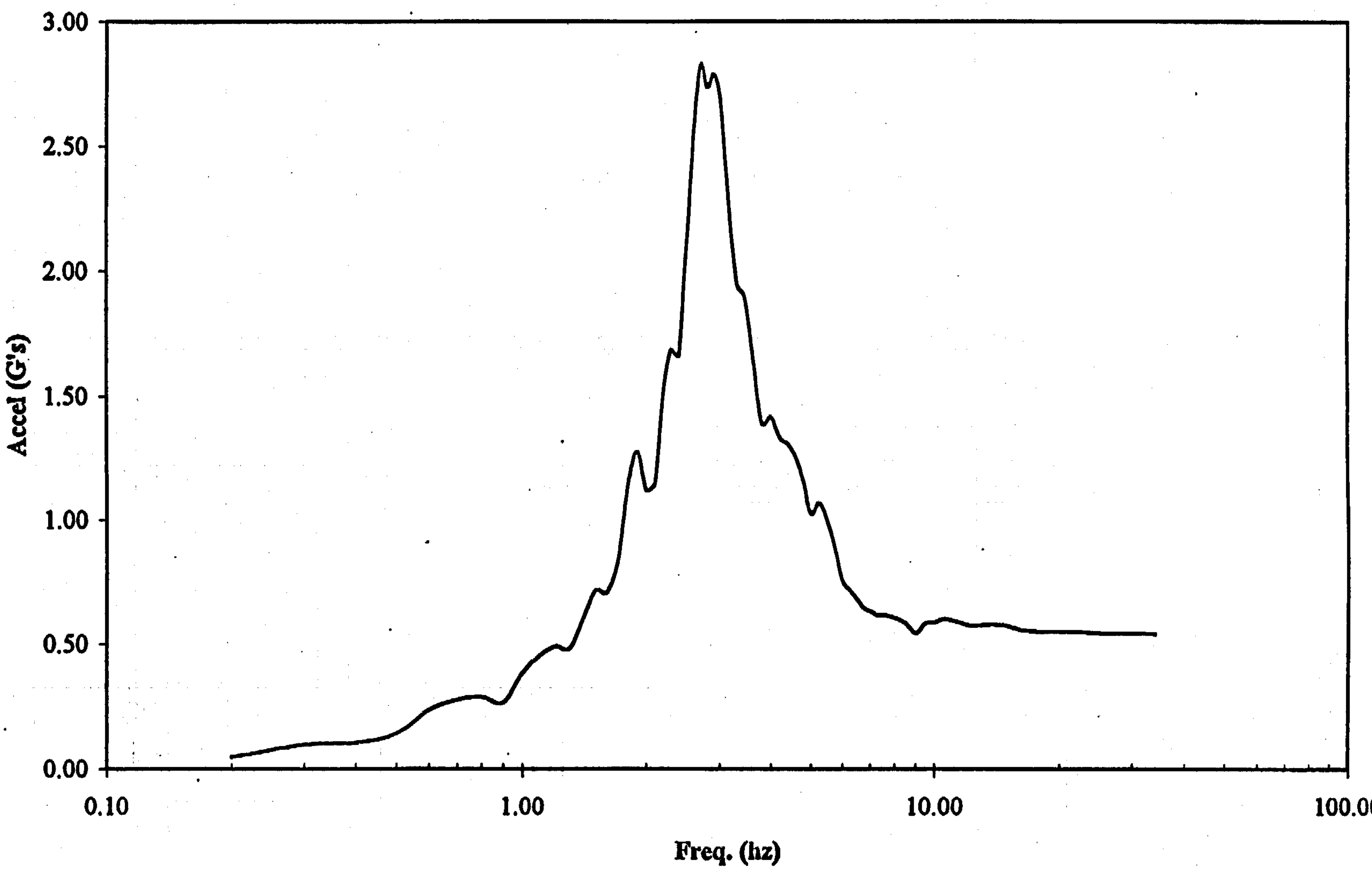


BM3 Input Spectrum (1%)

Occurence Time of Spectral Accelerations of the BM3 Input Spectrum

1% Damping

Freq (hz)	Peak Accel (g)	Occ. at sec.	Freq (hz)	Peak Accel (g)	Occ. at sec.
0.20	0.06	6.64	4.80	1.36	7.34
0.30	0.13	12.28	5.00	1.31	12.76
0.40	0.13	8.49	5.25	1.69	12.62
0.50	0.20	12.43	5.50	1.27	12.61
0.60	0.35	10.49	5.75	1.04	12.59
0.70	0.39	11.04	6.00	0.76	12.59
0.80	0.37	10.96	6.25	0.76	7.32
0.90	0.41	12.40	6.50	0.69	7.32
1.00	0.76	9.20	6.75	0.70	12.60
1.10	0.64	12.09	7.00	0.74	7.19
1.20	0.59	12.82	7.25	0.70	7.17
1.30	0.91	12.27	7.50	0.67	7.34
1.40	1.03	5.94	7.75	0.66	7.31
1.50	1.46	10.12	8.00	0.61	7.34
1.60	0.95	12.41	8.50	0.75	7.31
1.70	0.91	4.45	9.00	0.60	12.59
1.80	1.61	4.67	9.50	0.69	7.34
1.90	1.92	4.62	10.00	0.61	7.33
2.00	1.57	8.82	10.50	0.70	7.35
2.10	1.18	7.68	11.00	0.59	7.32
2.20	2.65	5.47	11.50	0.61	7.33
2.30	2.85	7.14	12.00	0.56	7.33
2.40	3.26	7.68	12.50	0.59	7.34
2.50	4.47	9.17	13.00	0.59	7.34
2.60	4.75	12.92	13.50	0.59	7.34
2.70	5.29	11.31	14.00	0.58	7.34
2.80	7.44	9.41	14.50	0.59	7.33
2.90	4.27	8.27	15.00	0.58	7.33
3.00	4.61	8.26	16.00	0.55	7.33
3.15	4.13	9.90	17.00	0.56	7.33
3.30	3.96	12.43	18.00	0.55	7.33
3.45	4.05	9.19	20.00	0.55	7.34
3.60	2.44	7.52	22.00	0.55	7.34
3.80	2.09	7.84	25.00	0.54	7.34
4.00	2.29	7.51	28.00	0.54	7.33
4.20	1.52	12.25	31.00	0.54	7.34
4.40	1.34	7.35	34.00	0.54	7.33
4.60	1.37	7.35			



BM3 Input Spectrum (5%)

Occurence Time of Spectral Accelerations of the BM3 Input Spectrum

5% Damping

Freq (hz)	Peak Accel (g)	Occ. at sec.	Freq (hz)	Peak Accel (g)	Occ. at sec.
0.20	0.05	6.57	4.80	1.16	7.34
0.30	0.10	12.22	5.00	1.03	7.34
0.40	0.10	12.82	5.25	1.07	12.62
0.50	0.14	12.43	5.50	0.99	12.61
0.60	0.24	10.47	5.75	0.88	12.60
0.70	0.28	10.99	6.00	0.75	7.32
0.80	0.29	10.93	6.25	0.72	7.32
0.90	0.27	11.79	6.50	0.68	7.32
1.00	0.38	9.19	6.75	0.64	7.32
1.10	0.45	12.07	7.00	0.63	7.18
1.20	0.49	12.02	7.25	0.62	7.18
1.30	0.48	12.26	7.50	0.62	7.33
1.40	0.59	5.91	7.75	0.61	7.33
1.50	0.71	4.45	8.00	0.60	7.33
1.60	0.70	4.43	8.50	0.58	7.32
1.70	0.83	4.43	9.00	0.54	7.35
1.80	1.13	4.65	9.50	0.58	7.35
1.90	1.27	4.62	10.00	0.59	7.35
2.00	1.12	4.60	10.50	0.60	7.34
2.10	1.15	4.60	11.00	0.59	7.34
2.20	1.51	5.45	11.50	0.59	7.34
2.30	1.69	5.42	12.00	0.58	7.34
2.40	1.66	5.39	12.50	0.57	7.34
2.50	2.11	9.15	13.00	0.58	7.34
2.60	2.57	9.12	13.50	0.58	7.34
2.70	2.83	9.28	14.00	0.58	7.34
2.80	2.74	8.31	14.50	0.58	7.33
2.90	2.79	8.11	15.00	0.57	7.33
3.00	2.70	7.91	16.00	0.56	7.33
3.15	2.28	7.89	17.00	0.55	7.33
3.30	1.95	7.88	18.00	0.55	7.33
3.45	1.90	7.86	20.00	0.55	7.34
3.60	1.68	7.53	22.00	0.55	7.34
3.80	1.39	7.52	25.00	0.54	7.34
4.00	1.42	7.51	28.00	0.54	7.33
4.20	1.33	7.36	31.00	0.54	7.34
4.40	1.31	7.35	34.00	0.54	7.33
4.60	1.25	7.35			

APPENDIX G

EVALUATION OF LINDLEY-YOW METHOD

by

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APPENDIX G

Fundamental to the Lindley-Yow method is the premise that the response of a system may be expressed as the sum of two components:

1. An "in-phase" component, the instantaneous value of which is equal to the corresponding, static effect of the forcing function; and

2. A series of "out-of-phase" components, the characteristics of which depend on the characteristics both of the forcing function and of the system itself.

The maximum value of the total response is then determined by SRSS combination of the numerical values of the component maxima.

For the base-excited systems examined in the report, the in-phase component at any time t is proportional to the ground acceleration, $\ddot{y}(t)$, and the out-of-phase component for a given mode of vibration is proportional to the instantaneous pseudo-acceleration, $A(t)$, of a single-degree-of-freedom system having the natural frequency of that mode. The maximum values of $\ddot{y}(t)$ and $A(t)$ will henceforth be identified as $\ddot{y}_o$ and A_o, respectively.

The maximum response obtained by this approach is clearly larger than the maximum static response, and in recognition of this fact, the method is said to apply only to system frequencies for which the amplification factor of response, AF, defined as the ratio of the maximum dynamic to maximum static response, is equal to or greater than unity. This factor, which is effectively the ratio $A_o/\ddot{y}_o$, is, of course, the reciprocal of the factor α used in the report. The relatively high natural frequency for which AF = 1 is denoted in the following by the symbol f_h

For system frequencies $f \geq f_h$, the out-of-phase component of response is zero, and the in-phase component is indeed equal to the static response. This is true both of single-degree-of-freedom systems and multi-degree-of-freedom systems with fundamental natural frequencies greater than f_h. As duly noted in the report, a static analysis of the system in this case, or an analysis that considers only the in-phase component of response will yield the correct solution.

As the natural frequency of the system decreases below f_h, it is well known that the AF increases and, after reaching a peak, which depending on the degree of periodicity of the forcing function may be quite high, it decreases, returning to the unit value, and then attains even lower values. Let f_l be the natural frequency of the relatively low-frequency system for which AF returns to unity. According to the Lindley-Yow method, the response history of a single-degree-of-freedom system with such a frequency will be identical to that of the corresponding static response. Expressed differently, the temporal variation of the pseudo-acceleration A(t) will be the same in this case as that of the input acceleration $\ddot{y}(t)$. This conclusion is simply unacceptable, as precise analyses will reveal that the response of such a low-frequency system has no resemblance to the static response, but is dominated instead by oscillations of a frequency equal or close to the natural frequency of system.

Since the characteristics of the solution cannot be expected to change abruptly with a change in the natural frequency of the system, the Lindley-Yow method should also be inapplicable over a range of natural frequencies higher than but close to f_l. By contrast, the method should give quite satisfactory results over a range of natural frequencies close to but lower than f_h. Furthermore, the higher the amount of damping of the system, the lower will be, in the latter case, the contribution of the out-of-phase component of response compared to that of the in-phase component, and hence, the wider will be expected to be the range of applicability of the Lindley-Yow method.

For broad-banded response spectra of the type associated with the 1940 El Centro earthquake record, which Lindley and Yow utilized, the frequency f_l may indeed be significantly lower than the fundamental natural frequency of the secondary systems that are of interest in the present study. However, the base-input motion for such a system is typically dominated by oscillations of a frequency equal to or close to the fundamental natural frequency of the primary system; the associated response spectrum is sharply peaked; and the fundamental natural frequency of the secondary system may well be close to f_l, particularly when the primary system is comparatively stiff. It follows that the limitations of the Lindley-Yow method in the neighborhood of the low-frequency limit f_l are not of academic interest only.

I believe that these matters should be emphasized in the description of method, as I interpret the objective of Chapter II of the report to be to provide not merely a description of the various methods, but also some evaluation of them. Incidentally, Gupta's method is superior in this respect, and this should also be duly acknowledged.

And now, to what I regard to be a more important point. When defined by the displacements $\{ u(t) \}$ of the points of mass concentration relative to the moving base, the response of an n-degree-of-freedom system can be expressed in the form

$$\{ u(t) \} = \sum_{j=1}^{n} c_j \{\phi_j\} A_j(t) \tag{1}$$

Where $\{ \phi_j \}$ = the jth mode of the vibration of the system; c_j = the participation factor for that mode; and $A_j(t)$ = the instantaneous pseudo-acceleration of a single-degree-of-freedom system with a natural frequency equal to the jth natural frequency of the given system.

If $A_j(t)$ in Eq. 1 is expressed as

$$A_j(t) = \ddot{y}(t) + \Delta A_j(t) \tag{2}$$

that is, as the sum of the ground acceleration and an incremental pseudo-acceleration, $\Delta A_j(t)$, Eq. 1 can be rewritten as

$$\{ u(t) \} = \sum_{j=1}^{n} c_j \{ \phi_j \} [\ddot{y}(t) + \Delta A_j(t)] \tag{3}$$

or as

$$\{ u(t) \} = \{ u_{st}(t) \} + \sum_{j=1}^{n} c_j \{ \phi_j \} \Delta A_j(t) \tag{4}$$

where $\{ u_{st}(t) \}$ represents the instantaneous values of the static displacements, and the series expression defines the corresponding dynamic increments.

In the Lindley-Yow method, the maximum value of the incremental pseudo-acceleration for a given mode, denoted in the following simply as ΔA_o, is given by

$$\Delta A_o = \left(A_o^2 - \ddot{y}_o^2 \right)^{\frac{1}{2}} \tag{5}$$

This is not an exact definition of ΔA_0 The exact maximum is given by the peak value of the difference of $A(t)$ and $\ddot{y}(t)$, and generally occurs at a time different from that for which $A(t)$ attains its maximum value.

An indication of the difference between the exact value of ΔA_o and that determined from Eq. 5 is provided in Table 1 and Fig. 1, which presents the relevant response spectra for undamped systems subjected to a relatively simple ground motion. The acceleration trace of the ground motion considered consists of a sequence of three half-sine pulses of alternating- signs and durations t_1, $2t_1$, and t_1, as shown in the inset diagram. The velocity and displacement traces of the motion are full-cycle and half-cycle pulses, respectively. The results are displayed in dimensionless form, with the pseudo-accelerations normalized with respect to the maximum input acceleration, and the abscissa representing the product of the natural frequency of the system and the duration of the middle acceleration pulse.

At high values of the frequency parameter $2ft_1$, the two sets of results in Fig. 1 both tend to a limiting value of zero. However, at the lower frequency values, they differ significantly to the left of the highest spectral peak. The very large differences in the results obtained in the low-frequency range raise serious questions about the applicability of either procedure in this range.

Neither procedure is logical for very-low-frequency systems. While I believe that it is possible to modify the procedure to yield satisfactory results in this frequency range as well, this extension is clearly beyond the scope of the present project. In this case, working with the deformation and pseudo-velocity values of the response spectra holds the secret, in my view, to improvement.

Table 1. Pseudo-acceleration values for undamped single-degree-of-freedom systems obtained by different procedures. Systems excited by a half-cycle displacement pulse.

$2ft_1$	$\sqrt{\left(\frac{A_o}{\ddot{y}_o}\right)^2 - 1}$	$\frac{\Delta A_o}{\ddot{y}_o}$
0.5	2.70	3.090
0.8	3.77	3.894
1.0	2.85	2.038
1.5	2.67	2.400
2.0	2.21	2.160
2.5	1.27	0.967
3.0	1.12	0.500

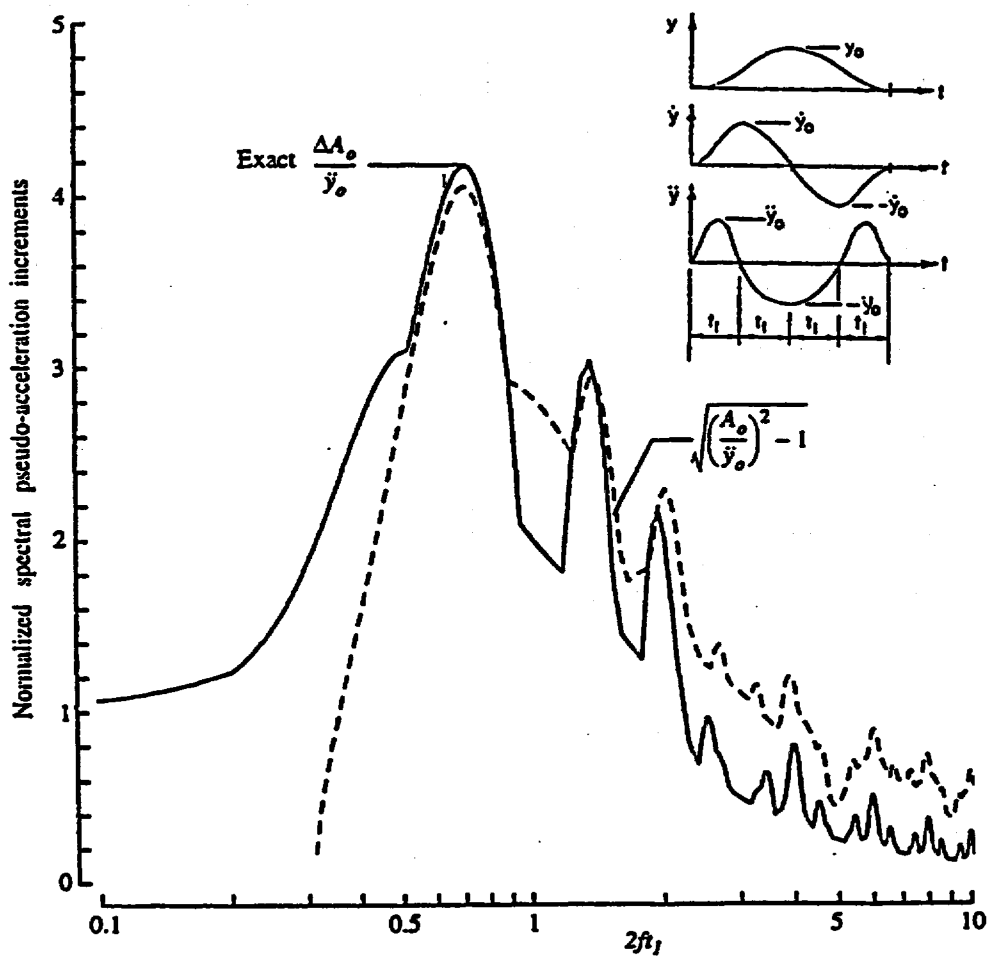

Fig. 1 Approximate and exact response spectra for incremental pseudo-acceleration of single-degree-of-freedom, undamped systems subjected to indicated base motion.

APPENDIX H

INDEPENDENT ANALYSIS OF BM3 PIPING MODEL

by

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APPENDIX H

Proposed Definition of f_2 for Gupta's Method

Unlike the RG 1.60 spectrum, the actual earthquake spectra do not have a sharp transition from the amplified to constant (ZPA) acceleration regions. To reduce the arbitrariness and in accordance with the Draft ASCE 4, we propose that the key frequency f_2 be defined as follows: "The key frequency f_2 is the minimum frequency at which spectra at various damping values converge." Although, there would still be some room for variable interpretation by different people, this definition is much more precise than the definition of f_r at which the spectral acceleration becomes equal to the ZPA. (In the Draft ASCE definition of f_2 and f_r have not been appropriately separated, f_r is assumed to be equal to f_2. This observation is true in the RG 1.60 spectra, but not in the spectrum at hand.)

For the input motion considered in this study, the above definition gives f_2 approximately equal to 6.0 Hz., as shown in the attached Fig. 1. We evaluated the values for rigid response coefficient, α, numerically for the input time history considered in this study and compared it with those evaluated using the definition of α given in section 2.2.3 of the report and f_2 = 6.0 Hz. As shown in the attached Fig. 2, the proposed definition of f_2 gives good estimation of α.

Independent Analysis of the BM3 Piping Model

We modeled the piping system BM3 on PIPESTRESS and performed a time history analysis using 14 modes and the missing mass effect. Table 1 compares the frequencies calculated by us (using PIPESTRESS) with those given in the report (using SAP). The two programs have different bend elements. Yet, the frequencies from the two sets of analyses are practically identical. Table 2 compares the time history results for x-direction input motion calculated by us with those given in the report. As shown in this table, the results from two sets of analyses are close to each other except for the reaction FY at node 1 for which the difference is 24%. Large difference in this reaction force may occur due to its insignificant value for the x direction input motion considered and effect of different bend stiffness.

Comparison of Gupta (f_2 = 6.0 Hz) and Lindley-Yow Methods to Time History Results for Three Directions of Input Motion

We performed response spectrum analyses using Lindley-Yow method and Gupta method with f_2 = 6.0 Hz. and using numerically calculated values of rigid response coefficient α. Tables 3 through 6 give the results for Lindley-Yow and Gupta methods and Tables 7 through 10 give the corresponding results for numerically calculated values of α.

We performed the time history and response spectrum analyses for all three directions of input motion. For vertical direction, the input motion was considered to be ⅔ of that in the horizontal direction. Table 6 compares the time history results with the response spectrum results for Lindley-Yow and Gupta method with $f_2 = 6.0$ Hz. The response values in this table are obtained by combining individual responses for input motions in x, y and z directions using SRSS rule. As shown in this table, the mean and standard deviation of responses from the two methods of combining modal responses are almost identical. Further, these mean and standard deviation values are practically the same as those calculated using numerically calculated value of α and given in Table 10.

Response tables 4-1 and 4-2 (end of Chapter 4 in the report) compare time history and response spectrum results for all the support reactions when input is applied in x-direction only. It is desirable to compare either only those reactions that get significant contribution from x-direction input analysis or all the reactions after combining responses from all the three (x, y and z) input motion analyses.

To illustrate the above, let us consider reactions MX at node 1, FY at node 23, and FX, FY and FZ at node 31. Table 11 gives the values of these reactions calculated from the time history and response spectrum analyses for each of the three directions of input motion and also the responses due to the three inputs combined by SRSS rule. As seen in this table, these reactions get most of their response from excitation in a particular direction. For excitation in other two directions, their values are insignificant. The insignificant values can have large differences in the time history and response spectrum results. However, the results from the two analyses are close to each other for significant response values and for the combined response.

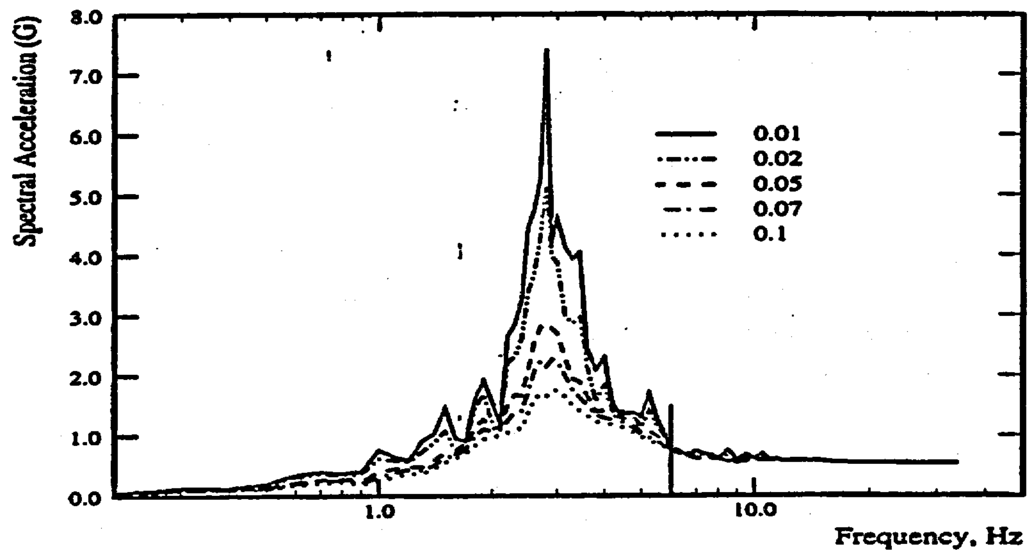

Figure 1: Input Response Spectra ($f_2 = 6.0$ Hz)

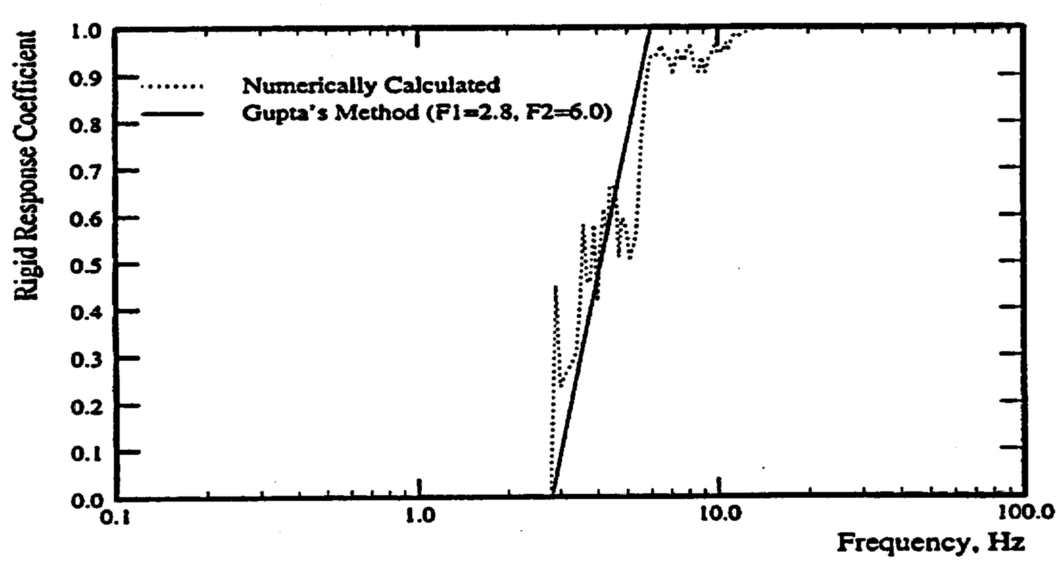

Figure 2: Variation of Rigid Response Coefficient with Frequency

Table 1: Frequencies (Hz) of BM3 Piping System

Mode	SAP S	PIPESTRESS P	Percent Difference (P-S)*100/S
1	2.9074	2.9074	0.00
2	4.3871	4.3871	0.00
3	5.5158	5.5158	0.00
4	5.7041	5.7042	0.00
5	6.9775	6.9776	0.00
6	7.3436	7.3434	0.00
7	7.8796	7.8794	0.00
8	10.302	10.3013	-0.01
9	11.056	11.0558	0.00
10	11.233	11.2329	0.00
11	11.498	11.4974	-0.01
12	12.431	12.4307	0.00
13	13.879	13.8790	0.00
14	16.122	16.1230	0.01

Table 2: Comparison of Time History Results from SAP and PIPESTRESS, X Direction Input

NODE NO.	REAC. TYPE	SAP (S)	PIPESTRESS (P)	Ratio P/S
1	FX	43.71	45.18	1.03
1	FY	4.36	3.32	0.76
1	FZ	1.6	1.65	1.03
1	MX	49.88	49.42	0.99
1	MY	776.4	783.27	1.01
1	MZ	278.42	273.95	0.98
4	FX	116.79	119.88	1.03
4	FZ	20.01	21.61	1.08
7	FY	13.27	13.75	1.04
11	FY	13.31	13.58	1.02
11	FZ	81.34	83.02	1.02
15	FX	731.47	743.50	1.02
17	FY	25.6	26.89	1.05
17	FZ	65.36	66.74	1.02
36	FY	46.69	48.92	1.05
36	FZ	42.12	41.82	0.99
38	FX	732.18	760.46	1.04
38	FY	43.44	43.35	1.00
38	FZ	29.95	29.93	1.00
38	MX	719.05	727.98	1.01
38	MY	2084.97	2086.88	1.00
38	MZ	3085.87	3078.87	1.00
23	FX	259.59	266.49	1.03
23	FY	26.08	25.94	0.99
31	FX	55.05	57.23	1.04
31	FY	14.17	14.17	1.00
31	FZ	16.08	16.61	1.03
31	MX	1137.3	1157.15	1.02
31	MY	612.38	622.11	1.02
31	MZ	1773.2	1794.08	1.01
Mean Value of 30 Comp.				1.01
Standard Dev. of 30 Comp.				0.05

F = Force (lbs)   M = Moment (in-lbs)

Table 3: Comparison of Time History and Response Spectrum Results, X Direction Input

NODE NO.	REAC. TYPE	Time-History (TH)	Gupta Method (GUP)	Ratio GUP/TH	Lindley-Yow (LY)	Ratio LY/TH
1	FX	45.18	45.43	1.01	46.21	1.02
1	FY	3.32	3.08	0.93	3.58	1.08
1	FZ	1.65	1.34	0.81	3.24	1.97
1	MX	49.42	17.06	0.35	112.68	2.28
1	MY	783.27	707.04	0.90	687.72	0.88
1	MZ	273.95	241.32	0.88	229.08	0.84
4	FX	119.88	109.30	0.91	104.60	0.87
4	FZ	21.61	16.22	0.75	33.36	1.54
7	FY	13.75	12.25	0.89	13.84	1.01
11	FY	13.58	11.16	0.82	15.00	1.10
11	FZ	83.02	71.77	0.86	69.80	0.84
15	FX	743.50	657.80	0.88	586.10	0.79
17	FY	26.89	25.36	0.94	35.81	1.33
17	FZ	66.74	55.83	0.84	63.26	0.95
36	FY	48.92	45.12	0.92	63.09	1.29
36	FZ	41.82	45.80	1.10	54.48	1.30
38	FX	760.46	776.20	1.02	768.90	1.01
38	FY	43.35	44.52	1.03	47.82	1.10
38	FZ	29.93	37.11	1.24	38.32	1.28
38	MX	727.98	507.96	0.70	901.20	1.24
38	MY	2086.88	2587.20	1.24	2672.40	1.28
38	MZ	3078.87	3157.20	1.03	3384.00	1.10
23	FX	266.49	326.10	1.22	343.30	1.29
23	FY	25.94	10.60	0.41	55.43	2.14
31	FX	57.23	56.06	0.98	56.71	0.99
31	FY	14.17	14.19	1.00	17.90	1.26
31	FZ	16.61	13.95	0.84	22.35	1.35
31	MX	1157.15	809.88	0.70	1636.80	1.41
31	MY	622.11	576.24	0.93	601.44	0.97
31	MZ	1794.08	1670.40	0.93	1759.20	0.98
Mean Value of 30 Comp.				0.90		1.22
Standard Dev. of 30 Comp.				0.19		0.36

F = Force (lbs) M = Moment (in-lbs)

* Gupta Method, with f_1=2.8 Hz and f_2=6.0 Hz

Table 4: Comparison of Time History and Response Spectrum Results, Y Direction Input

NODE NO.	REAC. TYPE	Time-History (TH)	Gupta Method (GUP)	Ratio GUP/TH	Lindley-Yow (LY)	Ratio LY/TH
1	FX	16.92	17.94	1.06	17.81	1.05
1	FY	110.20	96.04	0.87	98.51	0.89
1	FZ	3.99	3.43	0.86	4.25	1.07
1	MX	165.51	143.52	0.87	176.40	1.07
1	MY	138.03	148.56	1.08	158.28	1.15
1	MZ	1864.83	1497.60	0.80	1807.20	0.97
4	FX	24.46	22.70	0.93	26.93	1.10
4	FZ	21.55	15.96	0.74	32.19	1.49
7	FY	127.13	150.60	1.18	138.60	1.09
11	FY	199.14	202.70	1.02	189.50	0.95
11	FZ	24.15	17.93	0.74	34.97	1.45
15	FX	41.97	36.29	0.86	59.33	1.41
17	FY	97.96	102.70	1.05	97.60	1.00
17	FZ	13.05	7.83	0.60	16.25	1.24
36	FY	375.52	376.10	1.00	353.70	0.94
36	FZ	17.77	14.70	0.83	27.19	1.53
38	FX	11.12	7.18	0.65	14.65	1.32
38	FY	197.70	179.20	0.91	180.80	0.91
38	FZ	3.11	2.27	0.73	5.03	1.62
38	MX	2707.97	3094.80	1.14	2817.60	1.04
38	MY	211.07	153.72	0.73	336.48	1.59
38	MZ	6093.94	5581.20	0.92	5698.80	0.94
23	FX	20.51	21.07	1.03	26.87	1.31
23	FY	250.13	261.00	1.04	235.20	0.94
31	FX	3.62	1.31	0.36	6.39	1.76
31	FY	92.25	93.03	1.01	89.26	0.97
31	FZ	16.68	1.83	0.11	32.13	1.93
31	MX	2346.41	1747.20	0.74	2940.00	1.25
31	MY	327.44	314.64	0.96	325.08	0.99
31	MZ	673.64	568.44	0.84	752.52	1.12
Mean Value of 30 Comp.				0.86		1.20
Standard Dev. of 30 Comp.				0.22		0.27

F = Force (lbs) M = Moment (in-lbs)

* Gupta Method, with f_1=2.8 Hz and f_2=6.0 Hz

Table 5: Comparison of Time History and Response Spectrum Results, Z Direction Input

NODE NO.	REAC. TYPE	Time-History (TH)	Gupta Method (GUP)	Ratio GUP/TH	Lindley-Yow (LY)	Ratio LY/TH
1	FX	5.46	5.70	1.04	5.23	0.96
1	FY	16.42	16.80	1.02	14.23	0.87
1	FZ	36.40	33.29	0.91	39.89	1.10
1	MX	710.77	721.20	1.01	627.00	0.88
1	MY	462.39	344.40	0.74	652.44	1.41
1	MZ	508.10	520.80	1.02	430.20	0.85
4	FX	40.01	38.07	0.95	34.89	0.87
4	FZ	263.24	275.30	1.05	237.30	0.90
7	FY	28.26	29.06	1.03	32.37	1.15
11	FY	37.30	26.19	0.70	46.82	1.26
11	FZ	211.19	207.50	0.98	185.30	0.88
15	FX	69.48	62.23	0.90	89.16	1.28
17	FY	19.51	15.80	0.81	23.08	1.18
17	FZ	123.97	111.70	0.90	133.80	1.08
36	FY	29.18	30.50	1.05	30.17	1.03
36	FZ	882.28	882.90	1.00	856.30	0.97
38	FX	16.45	10.33	0.63	24.16	1.47
38	FY	8.21	8.03	0.98	11.29	1.38
38	FZ	265.01	251.70	0.95	255.20	0.96
38	MX	578.94	612.36	1.06	706.92	1.22
38	MY	8476.37	7932.00	0.94	8162.40	0.96
38	MZ	580.75	567.96	0.98	794.52	1.37
23	FX	44.83	37.37	0.83	44.20	0.99
23	FY	23.43	12.55	0.54	36.68	1.57
31	FX	16.34	15.08	0.92	17.43	1.07
31	FY	15.81	2.92	0.18	24.89	1.57
31	FZ	150.11	156.00	1.04	146.80	0.98
31	MX	10204.29	10142.40	0.99	10062.00	0.99
31	MY	666.02	620.64	0.93	708.36	1.06
31	MZ	1876.89	1759.20	0.94	1969.20	1.05
Mean Value of 30 Comp.				0.90		1.11
Standard Dev. of 30 Comp.				0.18		0.21

F = Force (lbs) M = Moment (in-lbs)

* Gupta Method, with f_1=2.8 Hz and f_2=6.0 Hz

Table 6: Comparison of Time History and Response Spectrum Results, Combined Responses for All Three Input Motions

NODE NO.	REAC. TYPE	Time-History (TH)	Gupta Method (GUP)	Ratio GUP/TH	Lindley-Yow (LY)	Ratio LY/TH
1	FX	48.55	49.18	1.01	49.80	1.03
1	FY	111.47	97.55	0.88	99.60	0.89
1	FZ	36.66	33.49	0.91	40.25	1.10
1	MX	731.46	735.54	1.01	661.02	0.90
1	MY	919.98	800.37	0.87	961.09	1.04
1	MZ	1952.13	1603.83	0.82	1871.77	0.96
4	FX	128.73	117.95	0.92	113.51	0.88
4	FZ	265.00	276.24	1.04	241.79	0.91
7	FY	130.96	153.87	1.17	143.00	1.09
11	FY	203.06	204.69	1.01	195.77	0.96
11	FZ	228.20	220.29	0.97	201.07	0.88
15	FX	747.92	661.73	0.88	595.80	0.80
17	FY	103.44	106.96	1.03	106.49	1.03
17	FZ	141.39	125.12	0.88	148.89	1.05
36	FY	379.81	380.02	1.00	360.55	0.95
36	FZ	883.45	884.21	1.00	858.46	0.97
38	FX	760.72	776.30	1.02	769.42	1.01
38	FY	202.57	184.82	0.91	187.36	0.92
38	FZ	266.71	254.43	0.95	258.11	0.97
38	MX	2863.26	3195.43	1.12	3041.51	1.06
38	MY	8732.04	8344.69	0.96	8595.33	0.98
38	MZ	6852.21	6437.41	0.94	6675.26	0.97
23	FX	271.01	328.91	1.21	347.18	1.28
23	FY	252.56	261.52	1.04	244.41	0.97
31	FX	59.62	58.07	0.97	59.67	1.00
31	FY	94.66	94.15	0.99	94.38	1.00
31	FZ	151.94	156.63	1.03	151.93	1.00
31	MX	10534.33	10323.61	0.98	10609.74	1.01
31	MY	968.42	903.46	0.93	984.47	1.02
31	MZ	2682.39	2491.61	0.93	2745.69	1.02
Mean Value of 30 Comp.				0.98		0.99
Standard Dev. of 30 Comp.				0.08		0.09

F = Force (lbs) M = Moment (in-lbs)

* Gupta Method, with f_1=2.8 Hz and f_2=6.0 Hz

Table 7: Comparison of Time History and Response Spectrum Results with Numerically Calculated Values of Rigid Response Coefficients, α, X Direction Input

NODE NO.	REAC. TYPE	Time-History (TH)	Response Spectrum (RS)	Ratio RS/TH
1	FX	45.18	44.86	0.99
1	FY	3.32	3.45	1.04
1	FZ	1.65	1.84	1.12
1	MX	49.42	52.86	1.07
1	MY	783.27	707.04	0.90
1	MZ	273.95	273.84	1.00
4	FX	119.88	110.70	0.92
4	FZ	21.61	21.31	0.99
7	FY	13.75	12.71	0.92
11	FY	13.58	11.63	0.86
11	FZ	83.02	73.18	0.88
15	FX	743.50	659.90	0.89
17	FY	26.89	28.15	1.05
17	FZ	66.74	59.28	0.89
36	FY	48.92	48.74	1.00
36	FZ	41.82	44.23	1.06
38	FX	760.46	771.20	1.01
38	FY	43.35	44.31	1.02
38	FZ	29.93	35.63	1.19
38	MX	727.98	639.96	0.88
38	MY	2086.88	2486.40	1.19
38	MZ	3078.87	3141.60	1.02
23	FX	266.49	313.30	1.18
23	FY	25.94	28.42	1.10
31	FX	57.23	56.05	0.98
31	FY	14.17	14.26	1.01
31	FZ	16.61	15.69	0.94
31	MX	1157.15	1065.24	0.92
31	MY	622.11	585.60	0.94
31	MZ	1794.08	1694.40	0.94
Mean Value of 30 Comp.				1.00
Standard Dev. of 30 Comp.				0.09

F = Force (lbs) M = Moment (in-lbs)

Table 8: Comparison of Time History and Response Spectrum Results with Numerically Calculated Values of Rigid Response Coefficients, α, Y Direction Input

NODE NO.	REAC. TYPE	Time-History (TH)	Response Spectrum (RS)	Ratio RS/TH
1	FX	16.92	15.21	0.90
1	FY	110.20	104.60	0.95
1	FZ	3.99	3.84	0.96
1	MX	165.51	161.76	0.98
1	MY	138.03	131.76	0.95
1	MZ	1864.83	1872.00	1.00
4	FX	24.46	22.42	0.92
4	FZ	21.55	24.38	1.13
7	FY	127.13	120.80	0.95
11	FY	199.14	212.50	1.07
11	FZ	24.15	26.60	1.10
15	FX	41.97	41.55	0.99
17	FY	97.96	116.20	1.19
17	FZ	13.05	11.23	0.86
36	FY	375.52	366.20	0.98
36	FZ	17.77	18.73	1.05
38	FX	11.12	9.55	0.86
38	FY	197.70	178.50	0.90
38	FZ	3.11	3.17	1.02
38	MX	2707.97	2954.40	1.09
38	MY	211.07	213.12	1.01
38	MZ	6093.94	5530.80	0.91
23	FX	20.51	22.15	1.08
23	FY	250.13	266.80	1.07
31	FX	3.62	3.70	1.02
31	FY	92.25	91.40	0.99
31	FZ	16.68	18.05	1.08
31	MX	2346.41	2187.60	0.93
31	MY	327.44	320.88	0.98
31	MZ	673.64	629.88	0.94
Mean Value of 30 Comp.				1.00
Standard Dev. of 30 Comp.				0.08

F = Force (lbs) M = Moment (in-lbs)

Table 9: Comparison of Time History and Response Spectrum Results with Numerically Calculated Values of Rigid Response Coefficients, α, Z Direction Input

NODE NO.	REAC. TYPE	Time-History (TH)	Response Spectrum (RS)	Ratio RS/TH
1	FX	5.46	4.82	0.88
1	FY	16.42	14.54	0.89
1	FZ	36.40	34.94	0.96
1	MX	710.77	703.08	0.99
1	MY	462.39	426.48	0.92
1	MZ	508.10	429.24	0.84
4	FX	40.01	36.14	0.90
4	FZ	263.24	265.60	1.01
7	FY	28.26	26.79	0.95
11	FY	37.30	37.08	0.99
11	FZ	211.19	200.10	0.95
15	FX	69.48	64.63	0.93
17	FY	19.51	16.83	0.86
17	FZ	123.97	119.40	0.96
36	FY	29.18	31.71	1.09
36	FZ	882.28	876.40	0.99
38	FX	16.45	14.91	0.91
38	FY	8.21	8.53	1.04
38	FZ	265.01	252.50	0.95
38	MX	578.94	642.00	1.11
38	MY	8476.37	7981.20	0.94
38	MZ	580.75	601.92	1.04
23	FX	44.83	39.56	0.88
23	FY	23.43	23.76	1.01
31	FX	16.34	15.81	0.97
31	FY	15.81	13.81	0.87
31	FZ	150.11	153.70	1.02
31	MX	10204.29	10140.00	0.99
31	MY	666.02	648.00	0.97
31	MZ	1876.89	1825.20	0.97
Mean Value of 30 Comp.				0.96
Standard Dev. of 30 Comp.				0.06

F = Force (lbs) M = Moment (in-lbs)

Table 10: Comparison of Time History and Response Spectrum Results with Numerically Calculated Values of Rigid Response Coefficients, α, Combined Responses for All Three Input Motions

NODE NO.	REAC. TYPE	Time-History (TH)	Response Spectrum (RS)	Ratio RS/TH
1	FX	48.55	47.61	0.98
1	FY	111.47	105.66	0.95
1	FZ	36.66	35.20	0.96
1	MX	731.46	723.38	0.99
1	MY	919.98	836.15	0.91
1	MZ	1952.13	1940.00	0.99
4	FX	128.73	118.59	0.92
4	FZ	265.00	267.57	1.01
7	FY	130.96	124.39	0.95
11	FY	203.06	216.02	1.06
11	FZ	228.20	214.72	0.94
15	FX	747.92	664.36	0.89
17	FY	103.44	120.74	1.17
17	FZ	141.39	133.78	0.95
36	FY	379.81	370.79	0.98
36	FZ	883.45	877.72	0.99
38	FX	760.72	771.40	1.01
38	FY	202.57	184.11	0.91
38	FZ	266.71	255.02	0.96
38	MX	2863.26	3090.34	1.08
38	MY	8732.04	8362.25	0.96
38	MZ	6852.21	6389.19	0.93
23	FX	271.01	316.56	1.17
23	FY	252.56	269.36	1.07
31	FX	59.62	58.35	0.98
31	FY	94.66	93.53	0.99
31	FZ	151.94	155.55	1.02
31	MX	10534.33	10427.84	0.99
31	MY	968.42	930.48	0.96
31	MZ	2682.39	2568.87	0.96
Mean Value of 30 Comp.				0.99
Standard Dev. of 30 Comp.				0.07

F = Force (lbs) M = Moment (in-lbs)

Table 11: Comparison of Time History and Response Spectrum Results for Some Specific Reactions

NODE NO.	REAC. TYPE	X-Direction		Y-Direction		Z-Direction		Combined	
		TH	RS	TH	RS	TH	RS	TH	RS
1	MX	49.42	17.06	165.51	143.52	710.77	721.20	731.46	735.54
23	FY	25.94	10.60	250.13	261.00	23.43	12.55	252.56	261.52
31	FX	57.23	56.06	3.62	1.31	16.34	15.08	59.62	58.07
31	FY	14.17	14.19	92.25	93.03	15.81	2.92	94.66	94.15
31	FZ	16.61	13.95	16.68	1.83	150.11	156.00	151.94	156.63

F = Force (lbs) M = Moment (in-lbs)

TH – Time-History

RS – Rsponse Spectrum using Gupta Method with f_1=2.8 Hz and f_2=6.0 Hz

APPENDIX I

CALCULATION OF MISSING MASS CONTRIBUTION TO TOTAL RESPONSE

APPENDIX I

Mathematical descriptions of the "missing mass" contribution to total response are contained in report References 3, 4, and 10. Reference 3 presents a step-by-step, mechanistic approach. Reference 4 presents a more complete mathematical description, which provides additional insight. Reference 10 essentially incorporates the mathematical description of Reference 4. It is recommended that Reference 4, Section 3.4 be reviewed to attain an understanding of the procedure.

The following steps can be utilized to calculate the response contribution of all system modes of vibration with frequencies equal to or greater than f_{zpa}.

Each direction of earthquake input motion must be considered separately.

Step 1: Determine the modal responses only for those modes that have natural frequencies less than that at which the spectral acceleration approximately returns to the ZPA (f_{zpa}).

Step 2: For each degree of freedom (DOF) included in the dynamic analysis, determine the fraction of DOF mass included in the summation of all of the modes included in Step 1. This fraction d_i for each DOF i is given by:

$$d_i = \sum_{n=1}^{N} c_{n,j}\, \phi_{n,i} \tag{1}$$

where,

n	is the mode number (1, 2, ..., N)
N	is the total number of modes included in Step 1
$\phi_{n,i}$	is the eigenvector value for mode n and DOF i
j	is the direction of input motion
$c_{n,j}$	is the participation factor for mode n in the jth direction:

$$c_{n,j} = \frac{\{\phi_n\}^T [m] \{\delta_{ij}\}}{\{\phi_n\}^T [m] \{\phi_n\}} \tag{2}$$

where δ_{ij} is the Kronecker delta, which is one if DOF i is in the direction of the earthquake input motion and zero if DOF i is a rotation or not in the direction of the earthquake input motion. This assumes that the three orthogonal directions of earthquake input motion are coincident with the DOF directions.

Next, determine the fraction of DOF mass not included in the summation of these modes:

$$e_i = d_i - \delta_{ij} \tag{3}$$

Step 3: Higher modes can be assumed to respond in phase with the ZPA and, thus, with each other; hence, these modes are combined algebraically, which is equivalent to pseudo static response to the inertial forces from these higher modes excited at the ZPA. The pseudo static inertial forces associated with the summation of all higher modes for each DOF i are given by:

$$P_i = ZPA \times M_i \times e_i \tag{4}$$

where P_i is the force or moment to be applied at DOF i.
M_i is the mass or mass moment of inertia associated with DOF i.

The structure is then statically analyzed for this set of pseudo static inertial forces applied to all of the degrees of freedom to determine the maximum responses associated with high-frequency modes not included in Step 1.

This procedure requires the computation of individual modal responses only for lower-frequency modes. Thus, the more difficult higher-frequency modes need not be determined. The procedure ensures inclusion of all modes of the structural model and proper representation of DOF masses.

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10. SUPPLEMENTARY NOTES

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11. ABSTRACT *(200 words or less)*

Regulatory Guide 1.92, "Combining Modal Responses and Spatial Components in Seismic Response Analysis," was last revised in 1976. The objectives of this project were to re-evaluate the current regulatory guidance for combining modal responses in response spectrum analysis; evaluate recent technical developments; and recommend revisions to the regulatory guidance. In addition, Standard Review Plan Section 3.7.2, "Seismic System Analysis," was reviewed to identify related sections which may need to be revised. The objectives were addressed through a literature review of past studies, supplemented by analysis of a piping system model previously utilized in NUREG/CR-5627, "Alternate Modal Combination Methods in Response Spectrum Analysis."

This project evaluated (1) methods for separation of the in-phase and out-of-phase modal response components; (2) methods for combination of the out-of-phase modal response components; (3) the contribution of "missing mass"; and (4) the combination of the three elements of response to produce the total response. Numerical results from response spectrum analyses were compared to corresponding time history analysis results to assess the accuracy of the various combination methods tested.

During the course of the project, several insights relating to potential improvements in the methodology for seismic analysis were identified and documented. These include (1) improvements in correlation between mode superposition time history and direct integration time history; (2) use of response spectrum generation single degree of freedom oscillator responses to define the frequency above which modal responses are in-phase with the input time history; and (3) evaluation of the effects of potential differences in mass distribution used in static and dynamic analyses of a piping system.

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